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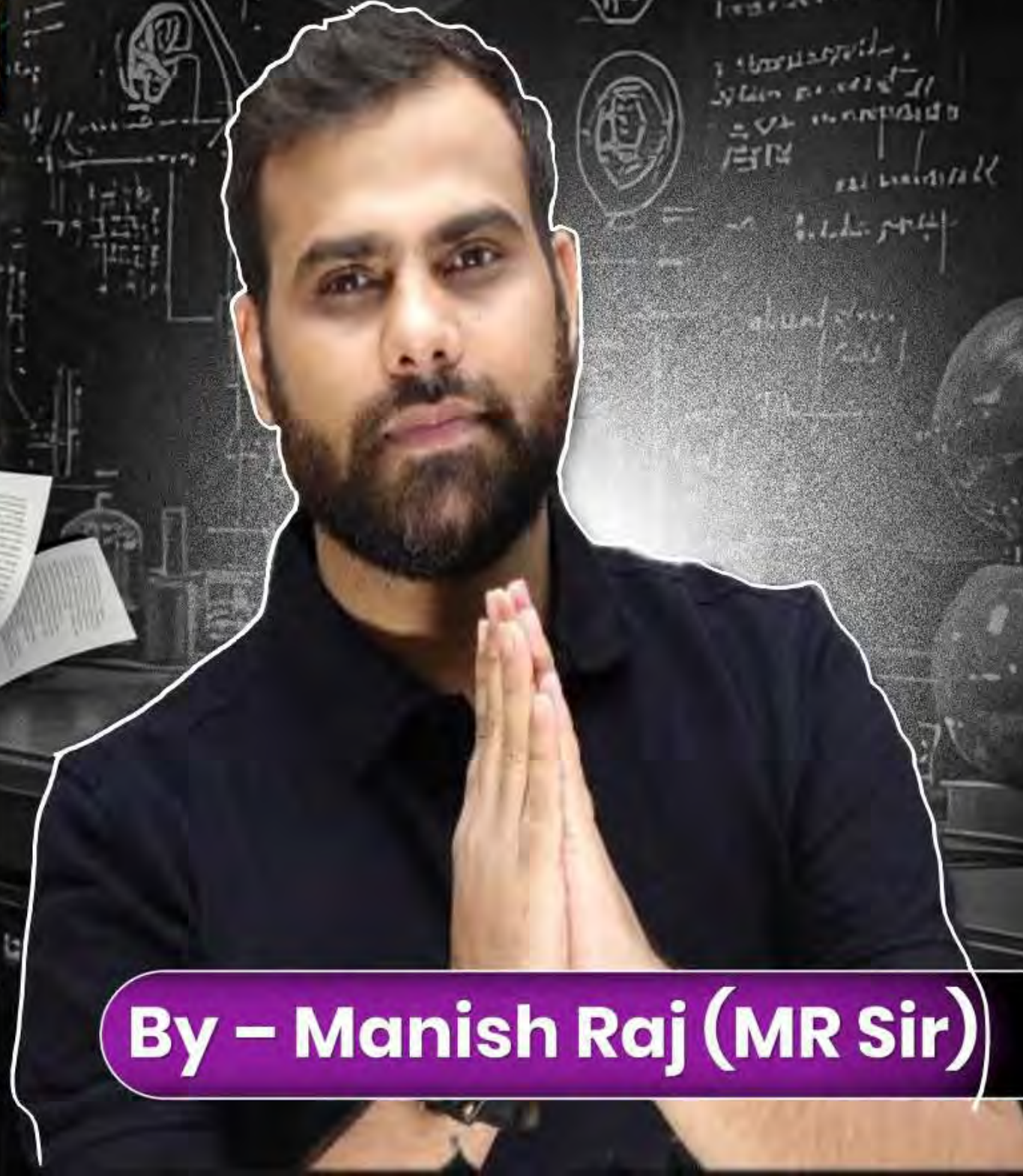
100

Centre of Mass and System of Particles

Physics

Lecture - 01

By – Manish Raj (MR Sir)

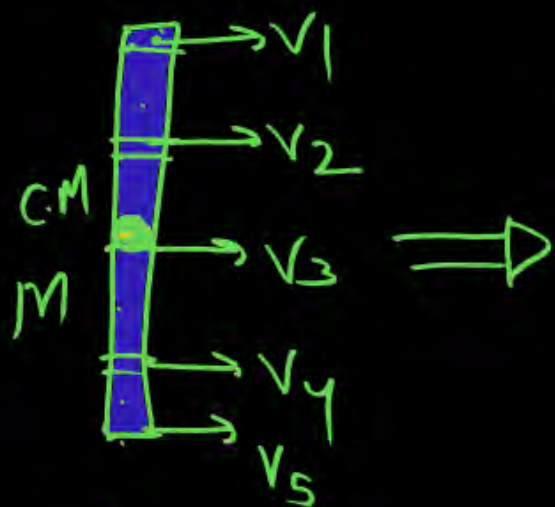


Today's Goal

Position of C.O.M., velocity of C.O.M.,
 a_{cm} of C.O.M.

$$\odot m \rightarrow \vec{v}$$

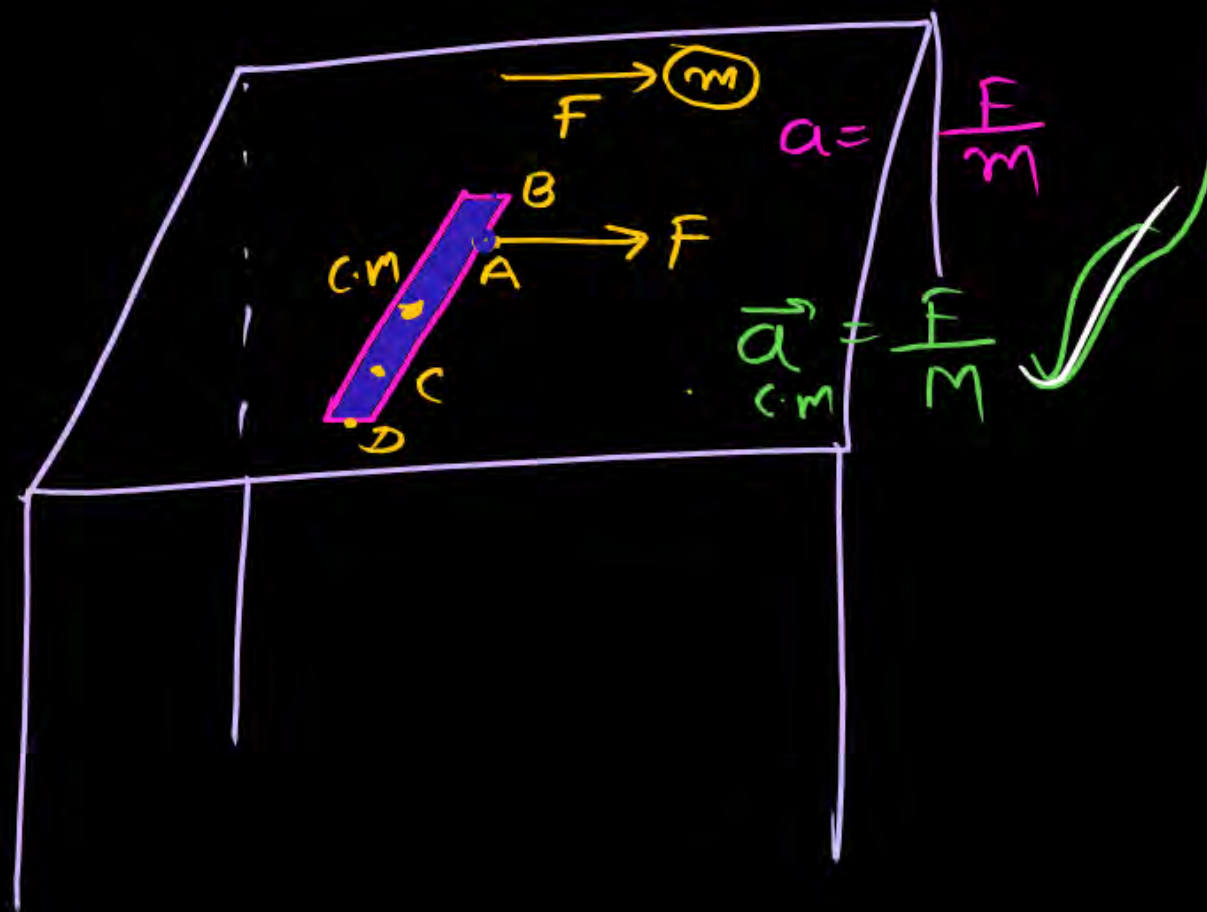
$$\vec{p} = m \vec{v} \quad \checkmark$$



Rod हल के point



$$\vec{p}_{\text{Rod}} = M \vec{v}_{\text{c.m.}} \quad \checkmark$$

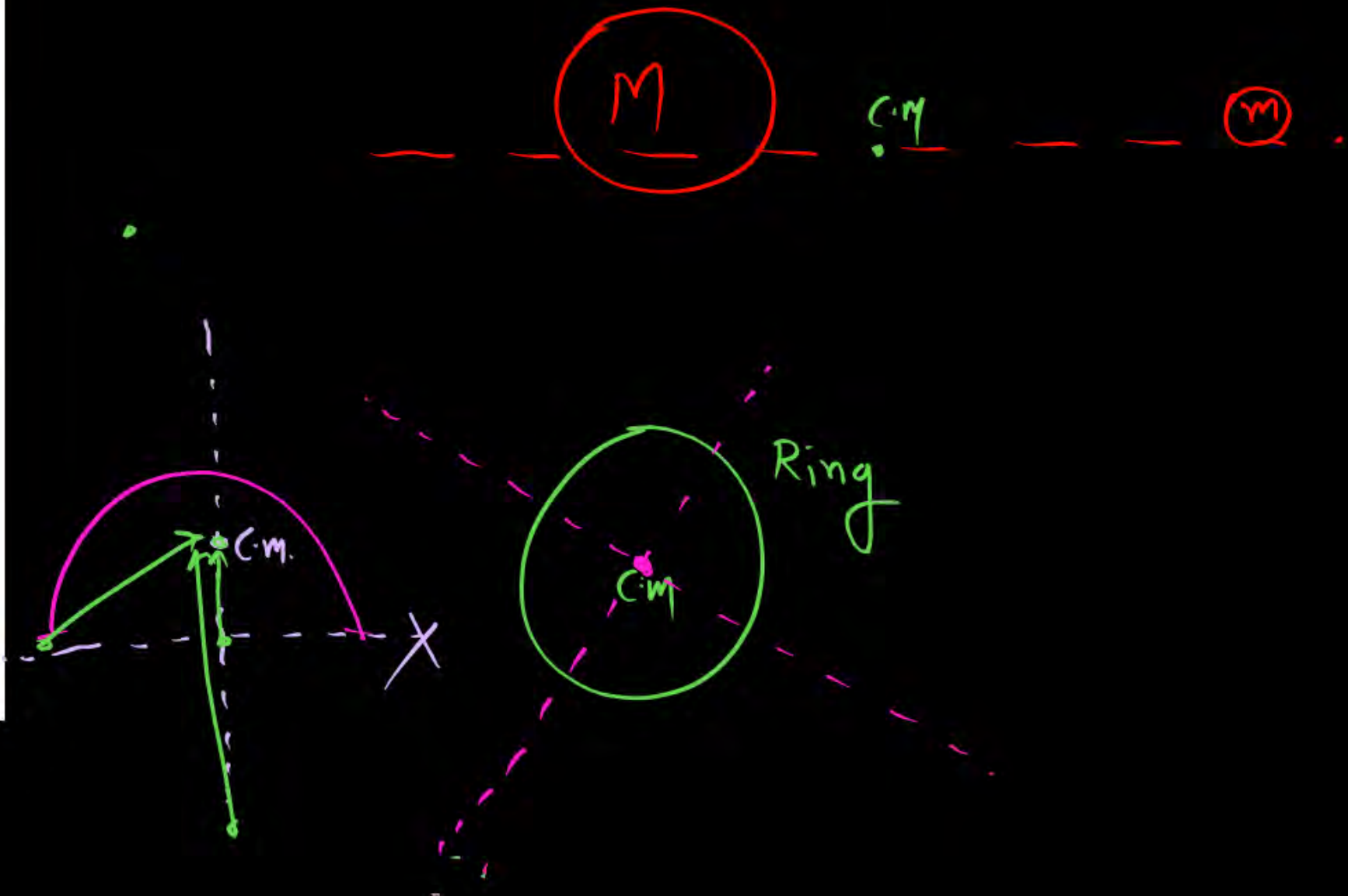


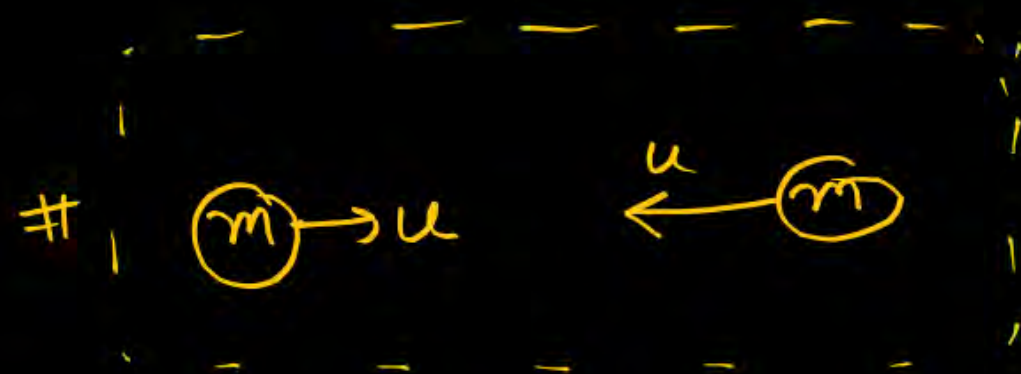
1 Centre of Mass

- A point where whole mass of the system may be assumed there; COM lies near to heavier object. ✓ ↑ for Translational Dynamics.
- It can be inside or outside the body.
- It lies always on the axis of symmetry and where two axis of symmetry will cut each other. ✓
- Position of COM depends upon frame of reference and choice of co-ordinates.
- ^{presence of} Centre of mass does not depend on choice of co-ordinate

MR* Box

- * Moment of inertia and K.E of system ke calculation ke liye C.O.M pe mass ko assume nahi karte. *





$$\vec{u}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$$\vec{v}_{cm} = \frac{m u \hat{i} - m u \hat{i}}{2m}$$

$$\vec{v}_{cm} = 0$$

$$P_{\text{system}} = m \vec{v}_{cm}$$

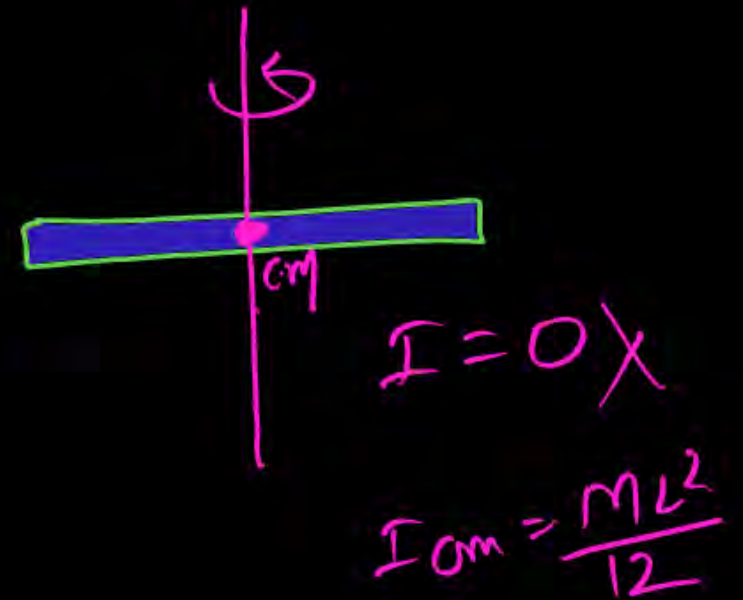
$$= 2M \times 0 = 0$$

$$K.E = \frac{1}{2} (2m) (0)^2 = 0$$

$$K.E = \frac{1}{2} m (u)^2 + \frac{1}{2} m (-u)^2$$

$$= \frac{1}{2} m u^2 + \frac{1}{2} m u^2$$

$$= \underline{m u^2}$$



C.O.M Ka concept system ka momentum & a.c.m nikalne
ke liye valid, hai

but K.E of system or Moment of Inertia of
system ke calculate ke liye C.O.M
ka concept valid nahi matlb
C.O.M pe marks assume kar ke
K.E_{sys} & M.O.I ka calculate nahi

Karna

Question



The centre of mass of a system of particles does not depend on

- 1 Masses of the particles ✗
- 2 Internal forces of the particles ✓
- 3 Position of the particles ✓
- 4 Relative distance between the particles ✓

Position of centre of mass of a body:

- 1 ☒ Depends on the choice of co-ordinate system
- 2 ☐ Is independent of the choice of co-ordinate system
- 3 ☐ May or may not depend on the choice of co-ordinate
- 4 ☐ None of the above

COM of discrete particle:-

$$\vec{r}_{cm} \text{ (Position of C.O.M.)} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3}$$

$$\vec{x}_{cm} = \frac{m_1 \vec{x}_1 + m_2 \vec{x}_2}{m_1 + m_2}$$

$$\vec{y}_{cm} = \frac{m_1 \vec{y}_1 + m_2 \vec{y}_2}{m_1 + m_2}$$

Motion of the centre of mass

Velocity of C.O.M

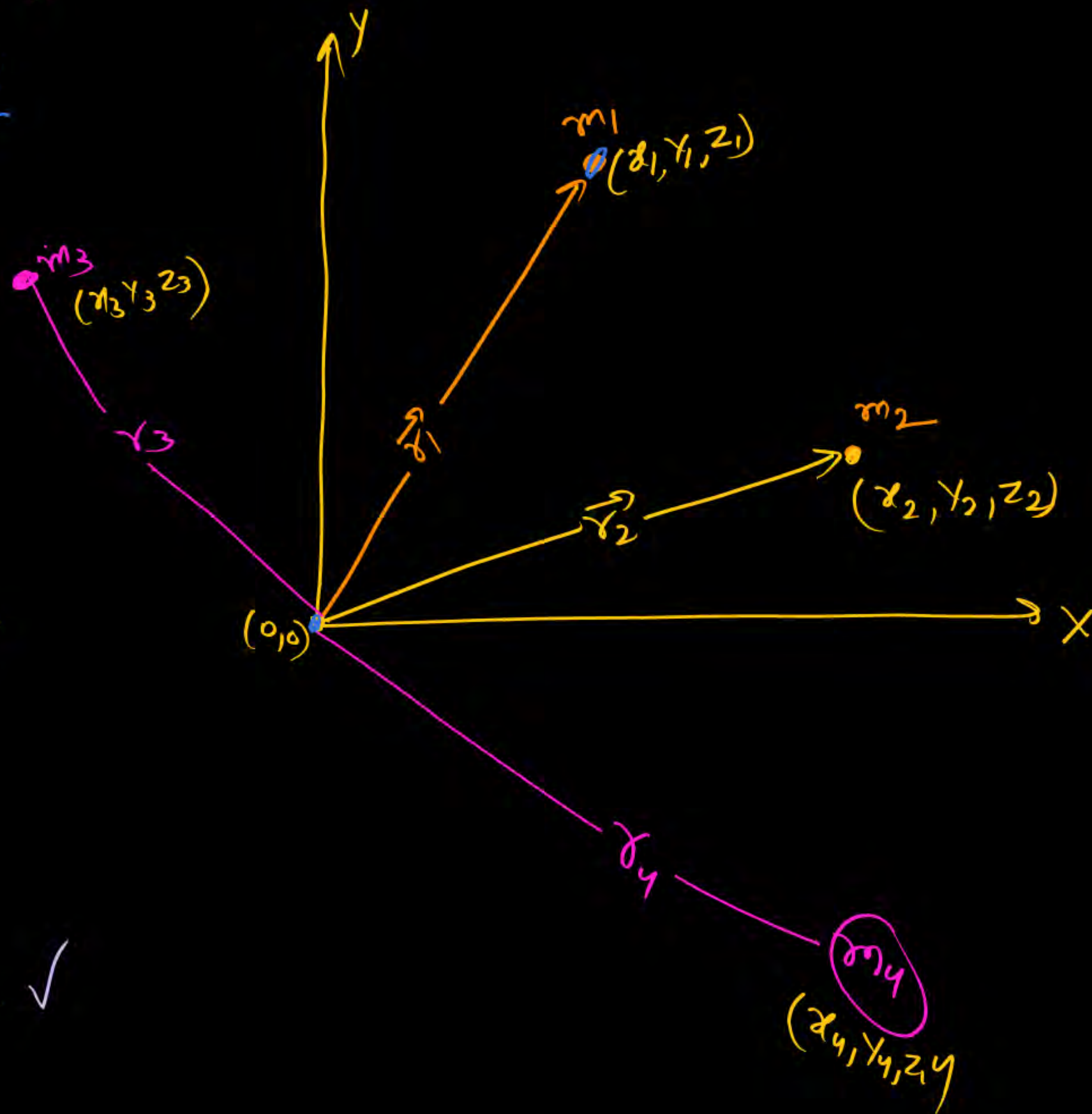
$$\vec{V}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

Acceleration of C.O.M

$$\vec{a}_{cm} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2}$$

MR*

$\vec{r}_{cm}, \vec{V}_{cm}, \vec{a}_{cm}$
ka formula
vector form
me hi use
karna hai.



$$\boxed{\vec{V}}_{cm} = \frac{m_1 \boxed{\vec{v}_1} + m_2 \boxed{\vec{v}_2}}{m_1 + m_2} \quad \checkmark$$

C.O.M of Two body system

(i) (ii)

$$\cancel{\gamma_{cm1}} = \frac{\cancel{m_2} \gamma}{\cancel{m_1 + m_2}}$$

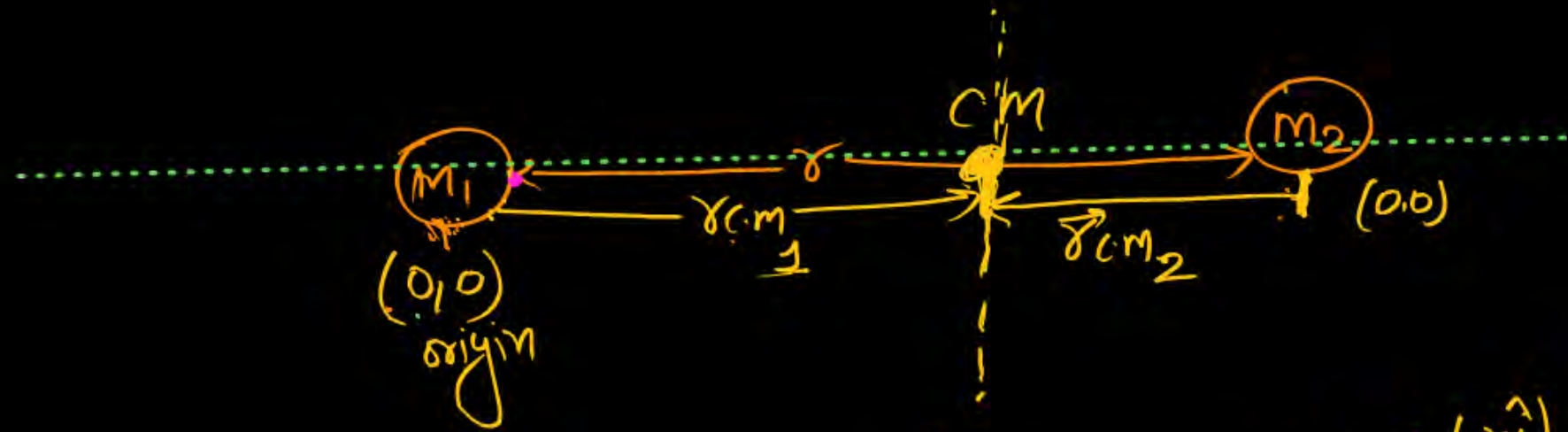
$$\cancel{\gamma_{cm2}} = \frac{\cancel{m_1} \gamma}{\cancel{m_1 + m_2}}$$

$$\frac{\gamma_{cm1}}{\gamma_{cm2}} = \frac{m_2}{m_1}$$

$$\boxed{m_1 \gamma_{cm1} = m_2 \gamma_{cm2}}$$

max

$$\frac{m_1 R_1}{R_1 + m_1}$$



$$\begin{cases} \vec{\gamma}_{cm} = \frac{m_1 \times 0 + m_2 \gamma \hat{i}}{m_1 + m_2} \\ \vec{\gamma}_{cm1} = \frac{m_2 \gamma \hat{i}}{m_1 + m_2} \end{cases} \quad \text{--- (i)}$$

$$\vec{\gamma}_{cm} = \frac{m_2 \times 0 + m_1 (-\gamma \hat{i})}{m_1 + m_2}$$

$$\vec{\gamma}_{cm2} = \frac{-m_1 \gamma \hat{i}}{m_1 + m_2} \quad \text{--- (ii)}$$

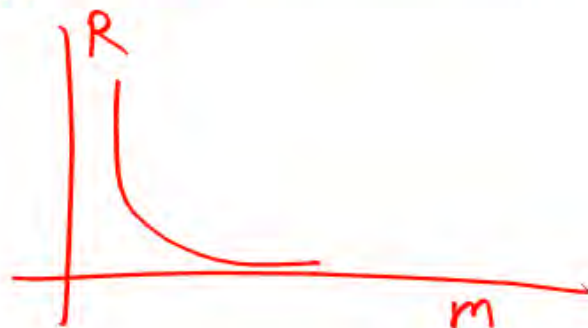
$\frac{m R^* \text{ fed}}{\text{gf } m_1 = m_2}$	$\frac{m R^* \text{ fed}}{\text{sf } m_2 = 0}$	gf $m_1 = 0$
$\vec{\gamma}_{cm} = \gamma/2 \hat{i}$	$\gamma = 0$	$\gamma_{cm} = \gamma$

Moment of Mass:

For two objects $MR = \text{constant}$.

- * This means that the product of mass and distance remains the same even if the individual values of mass and distance change.

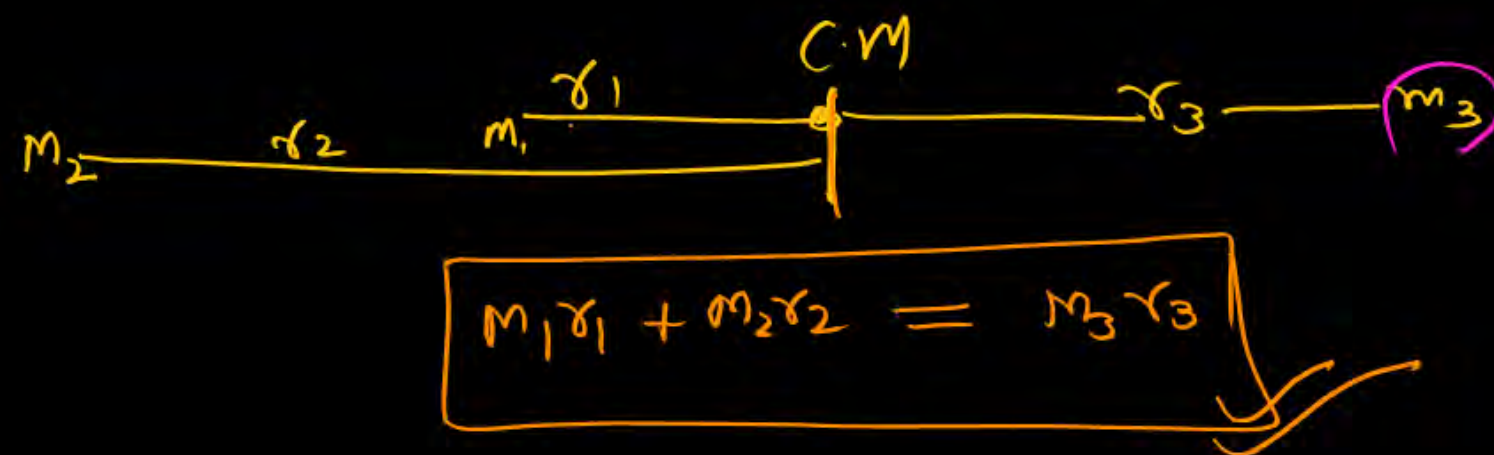
$$m \times R = \text{const}$$



- * Centre of Mass (Com) system ko equal moment of mass me divide karta hai.
- * Position, velocity, acceleration of centre of mass does not depends on internal force. ✓

MR* Box

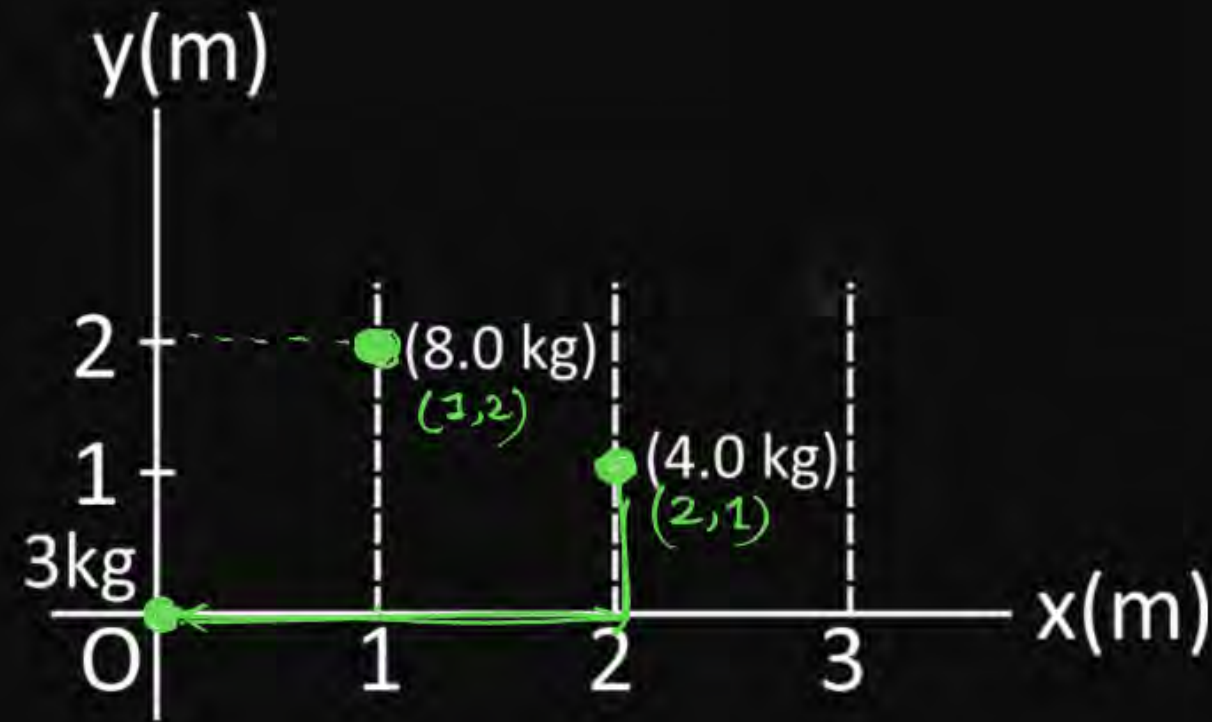
- * C.O.M. ka position nikalne ke liye sabse pehle sare object ka co-ordinate likho then use $\bar{r}_{cm}$.



Question

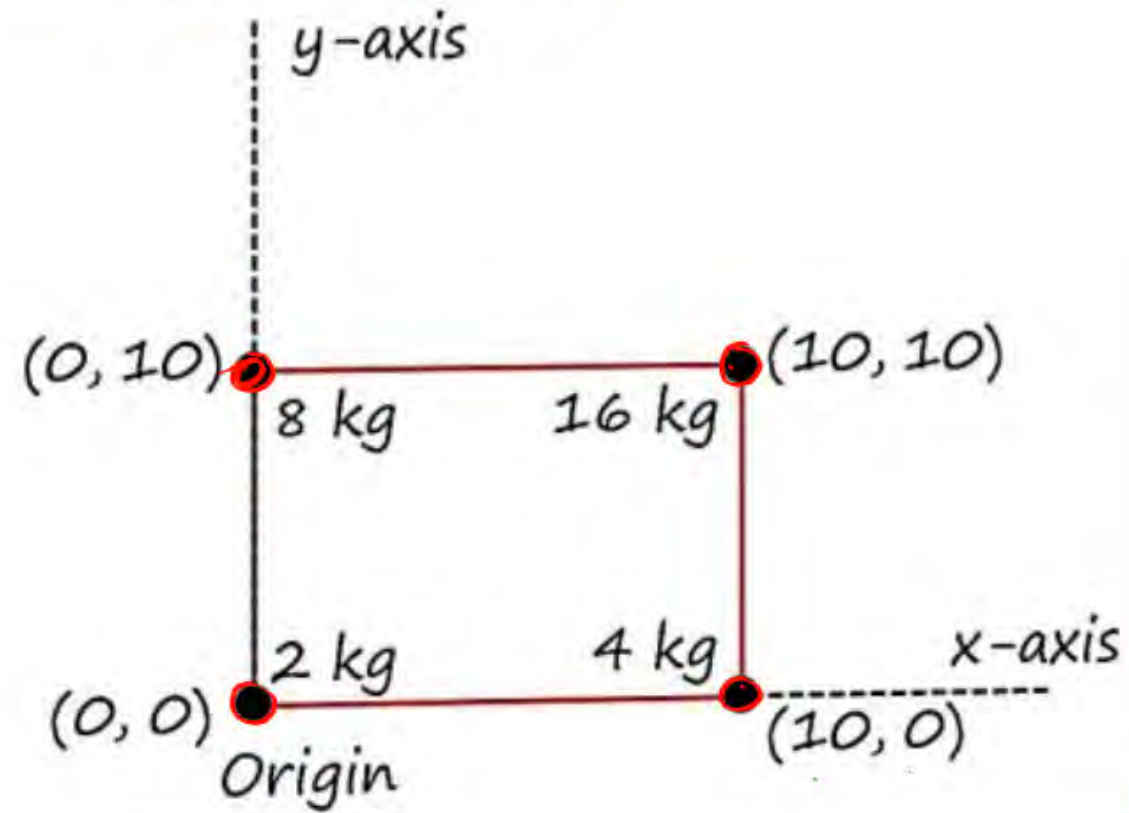


What are the coordinates of the centre of mass of the three particles system shown in figure?



$$\vec{r}_{cm} = \frac{3 \times 0 + 8(1\hat{i} + 2\hat{j}) + 4(2\hat{i} + 1\hat{j})}{3 + 8 + 4} = \frac{8\hat{i} + 16\hat{j} + 8\hat{i} + 4\hat{j}}{15} = \frac{16\hat{i} + 20\hat{j}}{15}$$

Q.1. Find position of C.O.M.?



$$\vec{r}_{c.m.} = \frac{2 \times 0 + 4(10\hat{i} + 0\hat{j}) + 16(10\hat{i} + 10\hat{j}) + 8(0\hat{i} + 10\hat{j})}{30}$$

Q.2. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-3\hat{i} - 2\hat{j} + \hat{k}$ respectively. The magnitude of position vector of centre of mass.

$$\text{Sol. } \vec{r}_{\text{c.m.}} = \frac{1(\hat{i} + 2\hat{j} + \hat{k}) + (-3\hat{i} - 2\hat{j} + \hat{k})}{4}$$

$$\vec{r}_{\text{c.m.}} = \frac{-2\hat{i} - 4\hat{j} + 2\hat{k}}{4}$$

$$\vec{r}_{\text{c.m.}} = -\hat{i} - \hat{j} + \hat{k}$$


Question



Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-3\hat{i} - 2\hat{j} + \hat{k}$ respectively. The magnitude of position vector of centre of mass of this system will be similar to the magnitude of vector:

[29 July, 2022]

- 1 $\hat{i} - 2\hat{j} + \hat{k}$
- 2 $-3\hat{i} - 2\hat{j} + \hat{k}$
- 3 $-2\hat{i} + 2\hat{k}$
- 4 $-2\hat{i} - \hat{j} + \hat{k}$ ✓

MCQ

If external force is zero then location of centre of mass will not change $\rightarrow$ False ✓

If $f_{\text{ext}} = 0$ then state of motion of COM will not change. $\left[\vec{V}_{\text{cm}} = \text{const}^n \right]$
 $a_{\text{cm}} = 0$ ✓

Shift in C.O.M.

$$\vec{\Delta r}_{c.m.} = \frac{m_1 \vec{\Delta r}_1 + m_2 \vec{\Delta r}_2}{m_1 + m_2}$$

If C.O.M. does not shift its position then

$$m_1 \vec{\Delta r}_1 = -m_2 \vec{\Delta r}_2$$

Δr_1 = shift in position of m_1

Δr_2 = shift in position of m_2

Δr_{cm} = shift in position of C.O.M. ✓

Q.14. Two blocks of masses 10 kg and 30 kg are placed along a vertical line. The first block is raised through a height of 7 cm. By what distance should the second mass be moved to raise the centre of mass by 1 cm?

Sol.
$$\Delta \vec{r}_{cm} = \frac{M_1 \Delta \vec{r}_1 + M_2 \Delta \vec{r}_2}{M_1 + M_2}$$

$$1 \text{ cm } \hat{j} = \frac{10 \times 7 \hat{j} + 30 \times \Delta r_2 \hat{j}}{40}$$
$$40 = 70 + 30 \Delta r_2$$
$$\Delta r_2 = -1 \text{ cm} //$$

2 Centre of mass of Continuous System

Area

Linear Mass Density (λ) for 1-Dimensional Systems:

$$\lambda = \frac{dm}{dL}$$

$$\int dm = \int \lambda dL$$

finite small mass $dm = \lambda dL$



Surface Mass Density (σ) for 2-Dimensional Systems:

$$\sigma = \frac{dm}{dA}$$

$$\int dm = \int \sigma dA$$

$$\text{mass} = \int \sigma dA$$

$$\text{Mass} = \sigma \times \text{Area}$$



Volume Mass Density ρ for 3-Dimensional Systems:

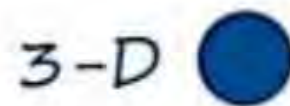
$$\rho = \frac{dm}{dV}$$

$$\int dm = \int \rho dV$$

$$\text{mass} = \int \rho dV$$

$\rho = \text{uniform}$

$$\text{Mass} = \rho \times \text{Volume}$$



$$M \propto \text{Volume}$$

$$M = \int \lambda dL$$

↑
Uniform

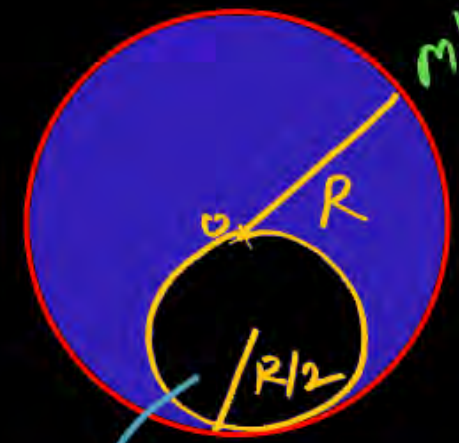
$$M = \lambda \int dL = (\lambda) L$$

Mass $\propto$ Length

$M, R = \text{Uniform}$



mass & Area
 $m \propto \pi R^2$



$$m'' = M - \frac{M}{4} = \frac{3M}{4}$$

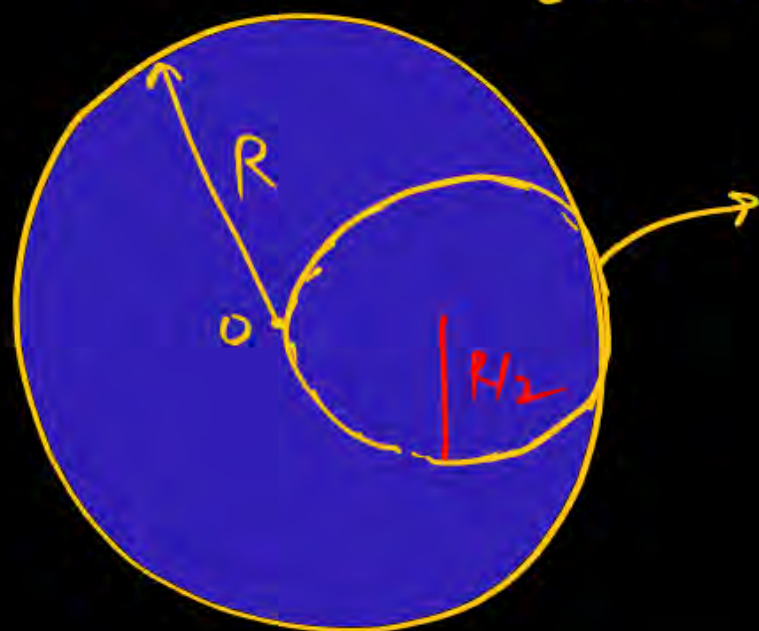


$$m'' = \frac{\pi R^2}{16} = \frac{m}{16}$$



$$m' \propto \pi \left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4} = \frac{M}{4}$$

Solid $\text{sp}^3(m, R)$
Uniform



$$M \propto \text{volume}$$
$$M \propto \frac{4}{3}\pi R^3$$

$$M' = \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 = \frac{\frac{4}{3}\pi R^3}{8} = \frac{M}{8}$$

$$M_{\text{rem}} = M - \frac{M}{8} = \frac{7M}{8} \checkmark$$

* To find the center of mass (x_{CM}) for a continuous system, we use the following integral formula:

$$x_{CM} = \frac{\int x \cdot dm}{\int dm}, \quad dm = \lambda \cdot dx$$

Q.4. Rod of length 'L' and Mass density $\lambda = \lambda_0 x$ then find position of C.O.M of Rod.

Sol.



$$dm = \lambda dx = \lambda_0 x dx$$

$$x_{CM} = \frac{\int x dm}{\int dm} = \frac{\int x \cdot \lambda_0 x dx}{\int \lambda_0 x dx}$$

MR** C.O.M of Rod:-

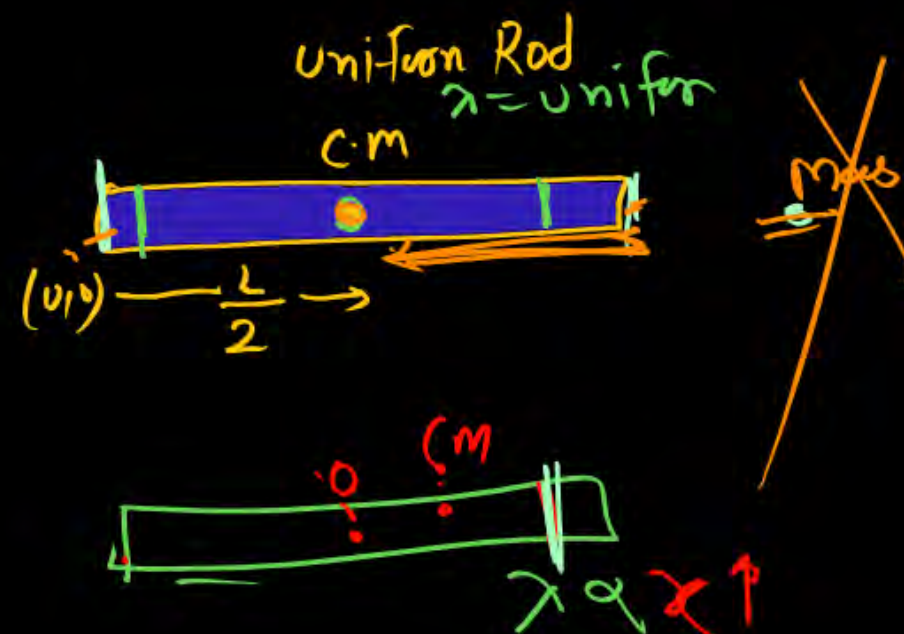
$$\left\{ \begin{array}{l} x_{CM} = L/2 \quad \lambda = \text{const}^n \\ x_{CM} > \frac{2L}{3} \quad \lambda \propto x \\ x_{CM} = \frac{3L}{4} \quad \lambda \propto x^2 \end{array} \right\}$$

$$\vec{r}_{CM} = \frac{\int \vec{r} dm}{\int dm}$$

$$\lambda = \lambda_0 x \leftarrow \text{Non-uniform}$$

$$= \frac{\lambda_0 \frac{x^3}{3} \Big|_0^L}{\lambda_0 \frac{x^2}{2} \Big|_0^L} = \frac{\frac{2}{3} (x)_0^L}{\frac{2}{2} (x)_0^L} = \frac{2L}{3}$$

$$x_{CM} = 0.66L$$



MR*

- + If density of rod varies linearly $\lambda = \lambda_0 x^n$ then position $\underline{L/2} < x_{cm} < \underline{L}$
- + If density of rod $\lambda = \alpha + \beta x$ then COM will be at $x_{cm} = L/2$ if $\beta = 0$

$\lambda = \alpha + \beta x \rightarrow$ when α & β are constⁿ

$\beta = 0$

$\lambda = \alpha = \text{unif}$

$x_{cm} = \underline{\underline{L/2}}$

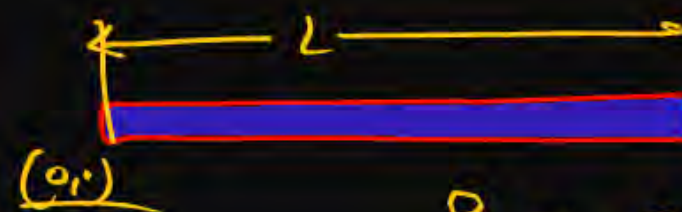
A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b \left(\frac{x}{L}\right)^2$ where, a and b are constants and $0 \leq x \leq L$. The value of x for the centre of mass of the rod is:
[9 Jan, 2020 (Shift-II)]

1 $\frac{4}{3} \left(\frac{a+b}{2a+3b} \right) L$ ✗

2 $\frac{3}{4} \left(\frac{2a+b}{3a+b} \right) L$ $\overset{MR^*}{=} \left(\frac{3 \times 2a}{4 \times 3a} \right) L = \frac{L}{2}$ ✓

3 $\frac{3}{2} \left(\frac{2a+b}{3a+b} \right) L$

4 $\frac{3}{2} \left(\frac{a+b}{2a+b} \right) L$



$$\rho = a + b \left(\frac{x}{L} \right)^2$$

$$\rho = a + \frac{bx^2}{L^2}$$

$$\boxed{MR^* \quad b=0}$$

$$\rho = a \rightarrow \text{const}$$

$$x = \frac{L}{2}$$

Question

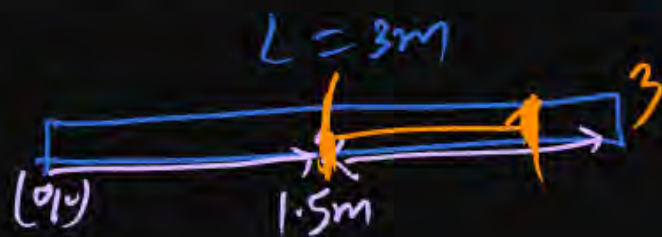
If linear density of a rod of length 3m varies as $\lambda = 2 + x$, then the position of the center of gravity of the rod is

1 $\frac{7}{3} \text{ m} = 2.3 \text{ m}$ ✗

2 $\frac{12}{7} \text{ m} = 1.7 \text{ m}$ ✓

3 $\frac{10}{7} \text{ m} = 1.4$ ✗

4 $\frac{9}{7} \text{ m}$ ✓



$\frac{m}{L}$ ✗

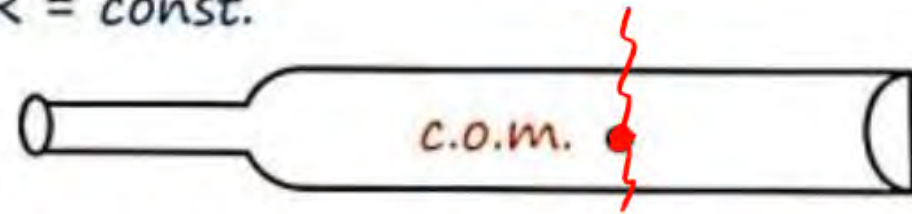
$1.5\text{m} < x < 3\text{m}$

$$x_{cm} = \frac{\int x dm}{\int dm}$$

$dm = \lambda dx$
 $\lambda = 2 + x$

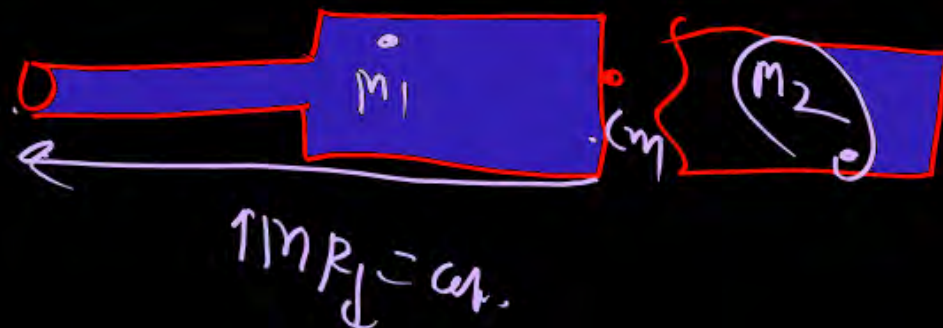
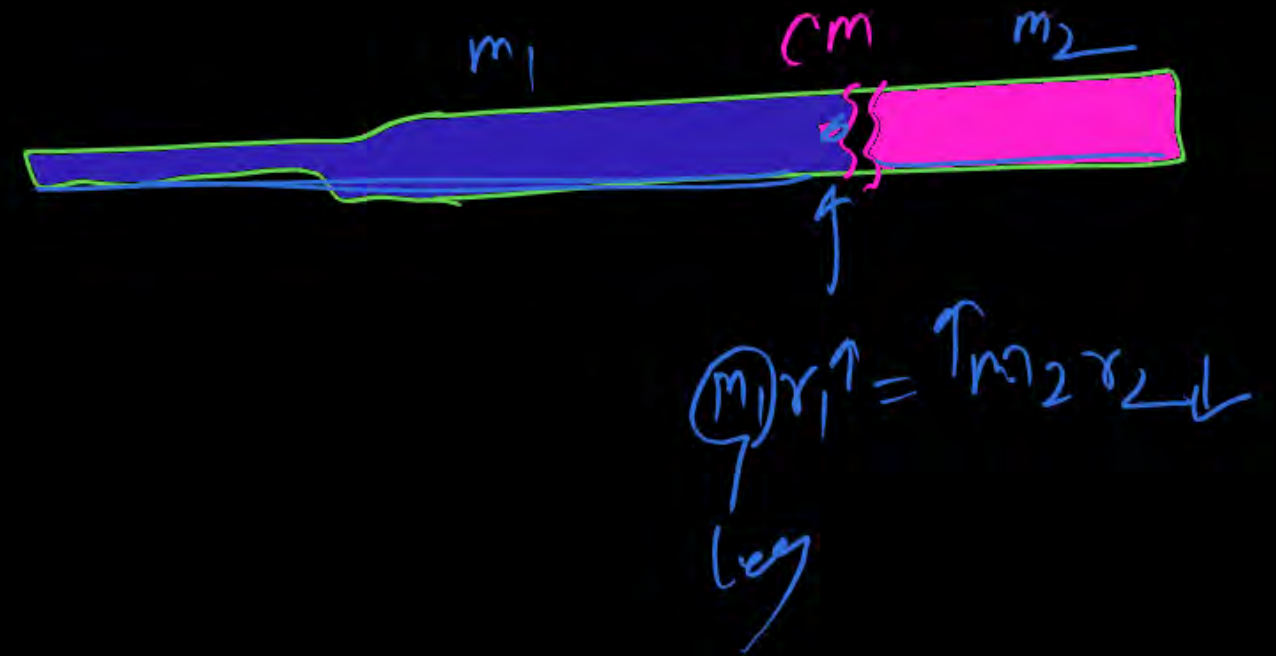
- + If we break cricket bat from c.o.m. then bottom part will have large mass, because c.o.m. divide system in two equal moment of mass.

$MR = \text{const.}$



Q.6. A cricket bat is cut at the location of its centre of mass as shown. Then

- ☒ (a) The Two pieces will have the same mass
- ☒ (b) The bottom piece will have large mass
- (c) The handle piece will have larger mass
- (d) Mass of handle piece is double the mass of bottom piece



Question

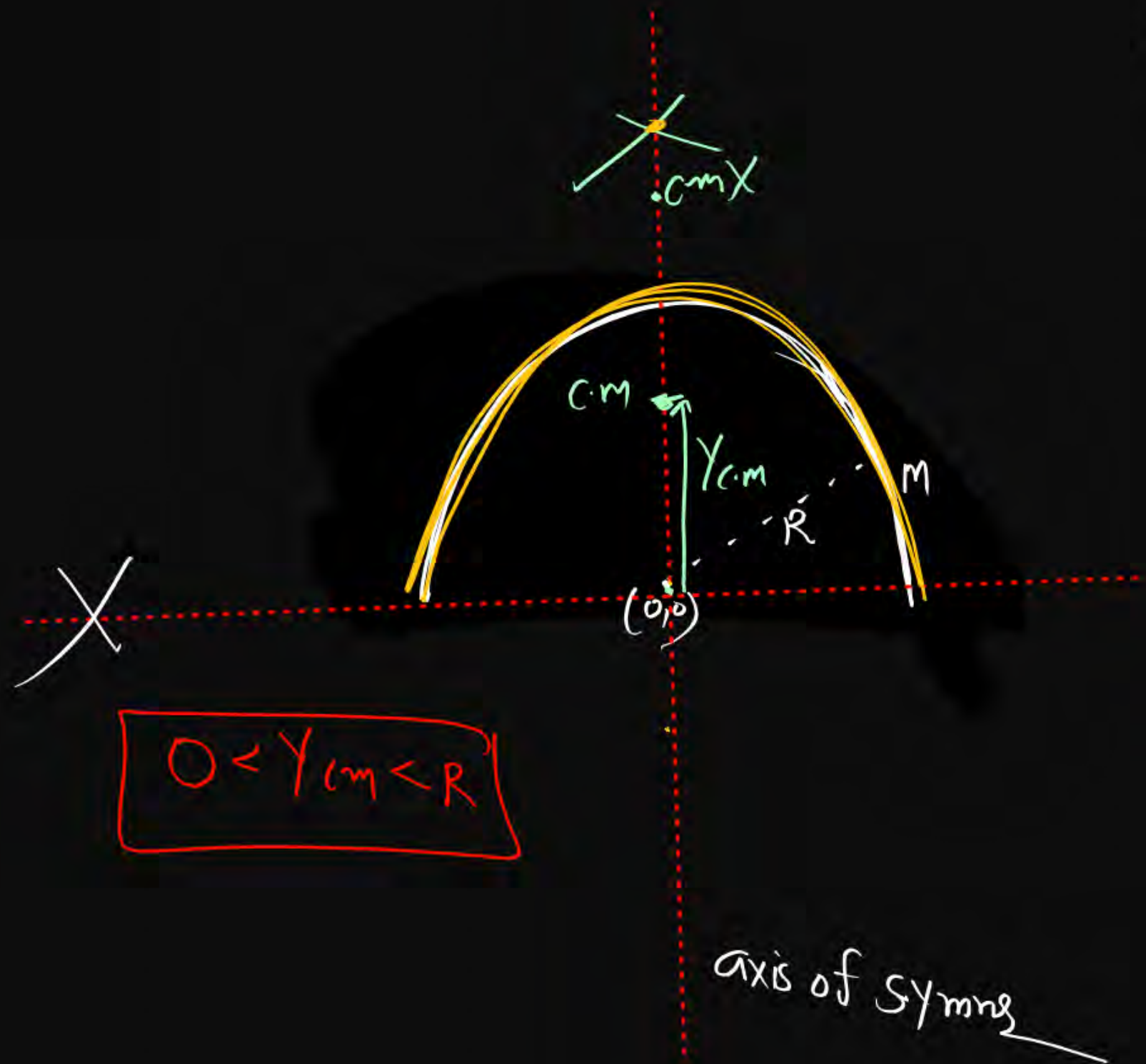
Find ^{Position} ~~acceleration~~ of centre of mass.

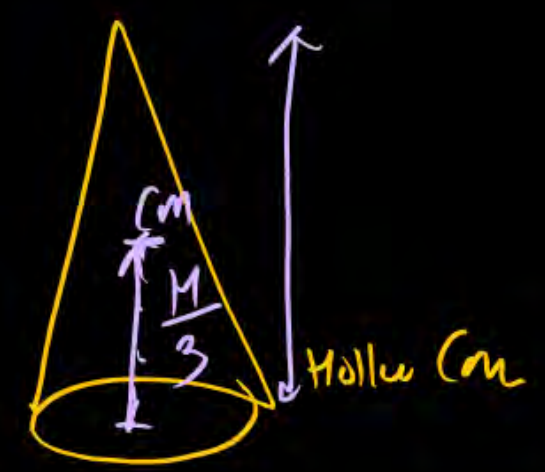
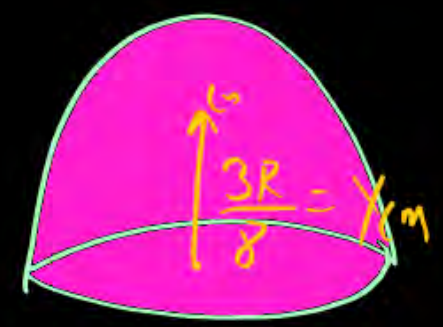
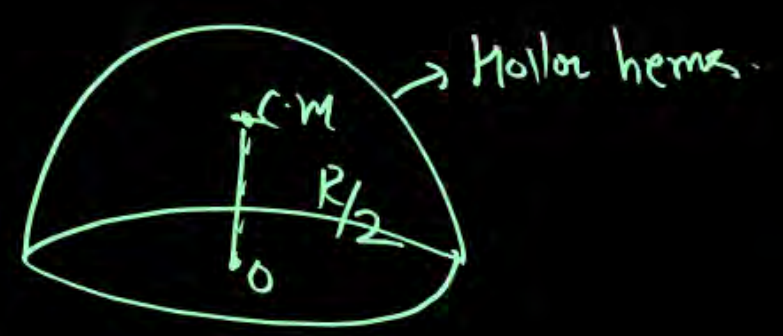
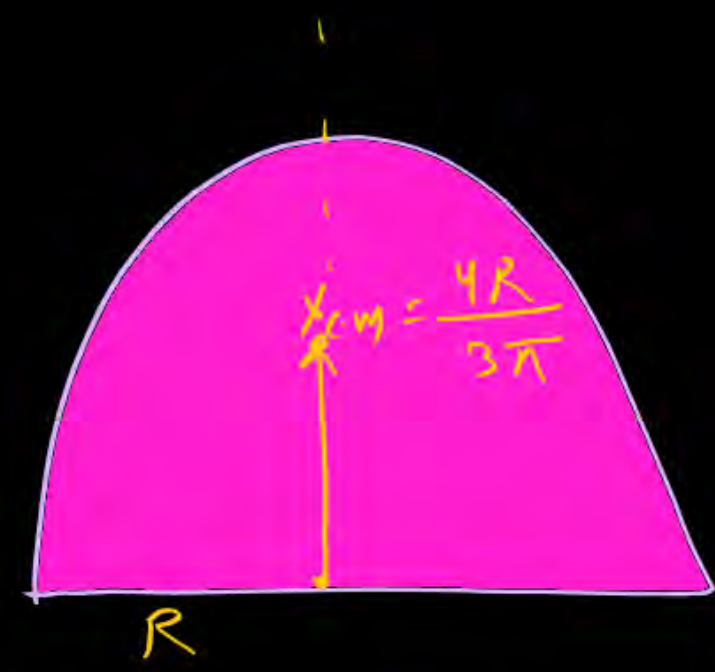
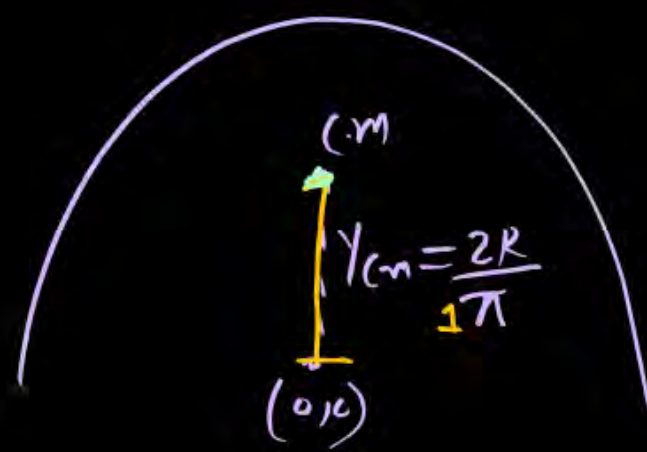
1 $\frac{2\pi}{R}$ ~~dim^n~~

2 $\frac{\pi R}{2} = \frac{3.14 R}{2}$ $Y_{c.m} = ?$ (m)

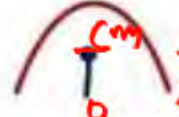
3 $\left[\frac{2R}{\pi} \right] = \frac{2 \times R}{3.14}$ ✓

4 $\frac{4R}{\pi} = \frac{4R}{3.14} = 1.1R$





Com of continuous mass system

Semi circular Ring  $y_c = 2R/\pi$

Solid Hemi sphere  $3R/8$

Semi circular Disc  $4R/3\pi$

Hollow Hemi sphere  $R/2$

Triangle / Hollow Cone.  $h/3$

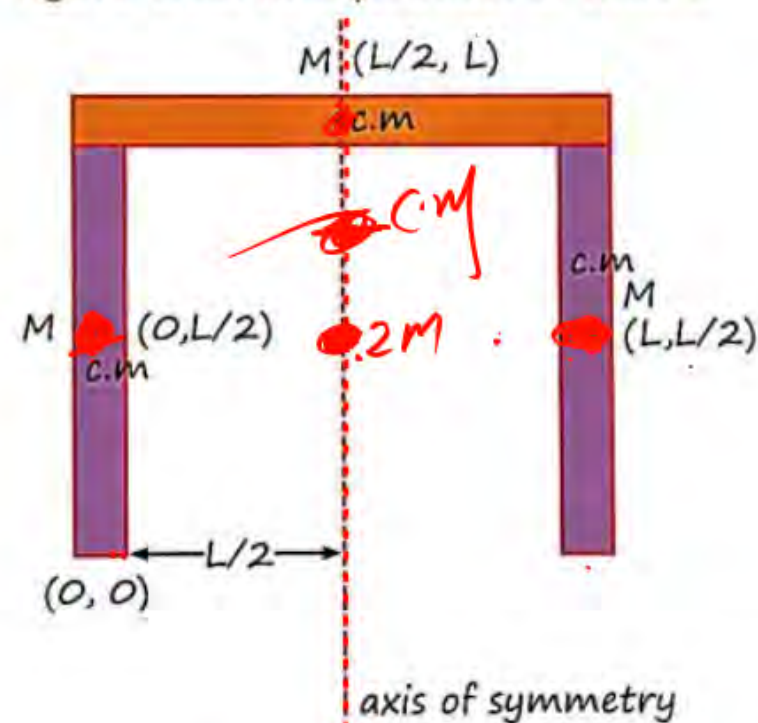
Cone Solid.  $y_c = h/4$

Circular Arc



$$y_{c.m.} = R \frac{\sin(\theta/2)}{\theta/2}$$

Q.8. Three identical rod (M, L) as shown in figure then find position of C.O.M.



$$\text{Sol. } \vec{r}_{\text{c.m.}} = \frac{M \frac{L}{2} \hat{j} + M \frac{L}{2} \hat{i} + mL \hat{j} + ML \hat{i} + \frac{ML \hat{j}}{2}}{3M}$$

$$= \frac{L}{2} \hat{i} + \frac{2L}{3} \hat{j}$$

MR* Box

+ Rod ko hatakar point object maan lo Rod ke CoM pe.

$$M \left(\frac{L}{2}, L \right)$$

$$M \left(0, \frac{L}{2} \right)$$

$$M \left(L, \frac{L}{2} \right)$$

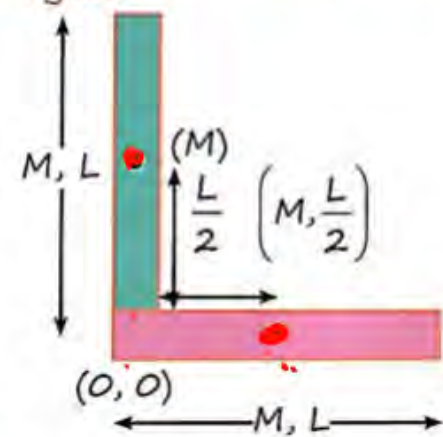
$$(0,0)$$

$$\vec{r}_{\text{c.m.}} = \frac{M \left(\frac{L}{2} \right) \hat{i} + M \left(\frac{L}{2} \hat{i} + L \hat{j} \right) + M \left(L \hat{i} + \frac{L}{2} \hat{j} \right)}{3M}$$

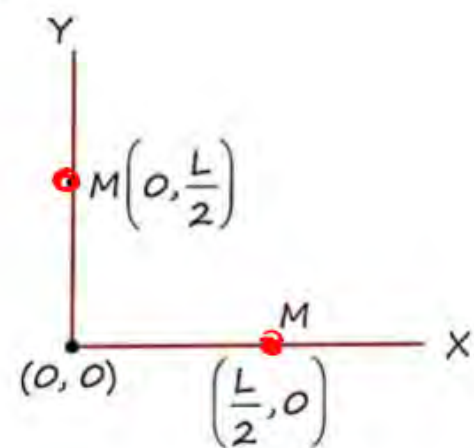
$$= \frac{\frac{3L}{2} \hat{i} + 2L \hat{j}}{3}$$

$$= \frac{L}{2} \hat{i} + \frac{2L}{3} \hat{j}$$

Q.7. Two identical rod (M, L), as shown in figure:-



Sol.



90

$$r_{cm} = \frac{M \frac{L}{2} \hat{i} + M \frac{L}{2} \hat{j}}{2M}$$

$$= \frac{L}{4} \left(\hat{i} + \hat{j} \right)$$

Question

A rod of length 5 L is bent right angle keeping one side length as 2 L. The position of the centre of mass of the system: (Consider $L = 10$ cm) **[07 April, 2025 (Shift-I)]**



1 $2\hat{i} + 3\hat{j}$

2 $3\hat{i} + 7\hat{j}$

3 $5\hat{i} + 8\hat{j}$

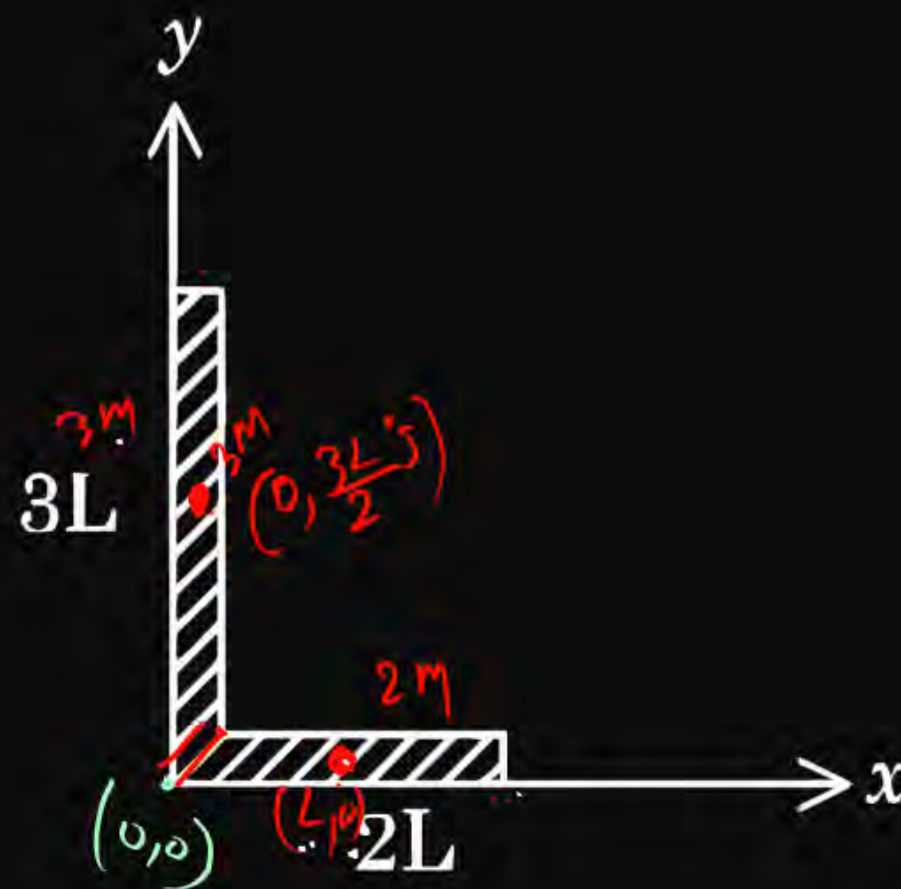
4 $4\hat{i} + 9\hat{j}$

$$r_{cm} = \frac{2m \times \hat{i} + 3m \times \frac{3L}{2} \hat{j}}{5m}$$

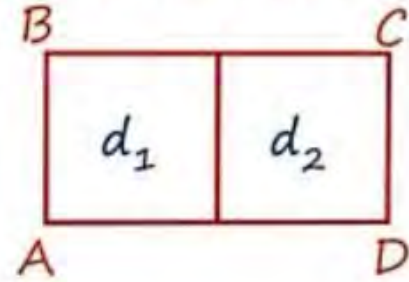
$$= \frac{2\hat{i} + \frac{9}{2}L\hat{j}}{5}$$

$$= \frac{2}{5}\hat{i} + \frac{9}{10}L\hat{j}$$

$$= 4\hat{i} + 9\hat{j}$$



Q.5. Half of the uniform rectangular plate of Length ' L ' is made up of material of density ' d_1 ' and the other half with density d_2 . The perpendicular distance of center of mass from AB is



Question



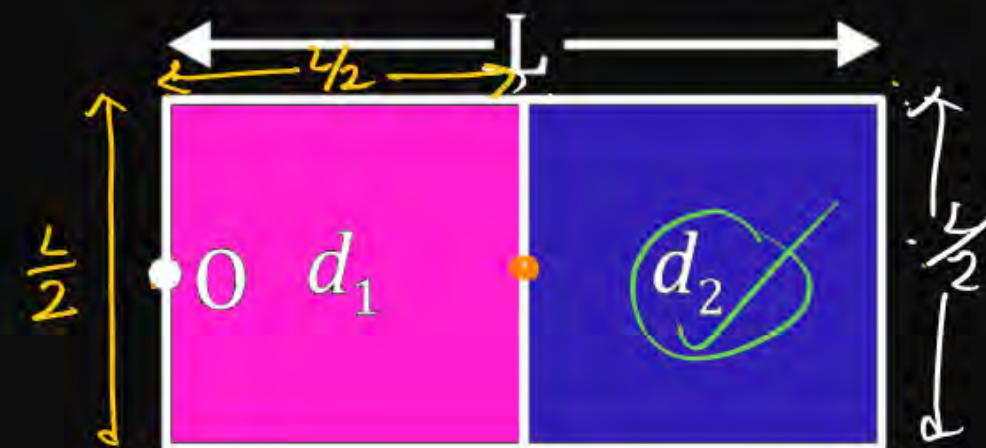
Shows a rectangular plane of length L , the half of which is made of material of density d_1 and another half of density d_2 . The distance of centre of mass from the origin O is

1 $\frac{d_1 + 2d_2}{2(d_1 + d_2)} L$ $\frac{3L}{4}$
MR*
gf $d_1 = d_2$

2 $\frac{(d_1 + 3d_2)L}{4(d_1 + d_2)}$ $x_m = \frac{L}{2}$
 $\frac{4L}{8} = \frac{L}{2}$

3 $\frac{(d_1 + 3d_2)L}{2(d_1 + d_2)}$ MR*
gf $d_2 = 0$
 $x_m = \frac{L}{4}$

4 $\frac{(3d_1 + d_2)L}{4(d_1 + d_2)}$ $\frac{4L}{8} = \frac{L}{2}$



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{d_1 \frac{L^2}{4} \times \frac{L}{4} + d_2 \frac{L^2}{4} \left(\frac{3L}{4} \right)}{d_1 \frac{L^2}{4} + d_2 \frac{L^2}{4}}$$

$$= \frac{(d_1 + 3d_2) \frac{L}{4}}{d_1 + d_2}$$

$(0,0)$ $\rightarrow$ $m_1 = d_1 \frac{L^2}{4}$ $\rightarrow$ $m_2 = d_2 \frac{L^2}{4}$

$\left(\frac{L}{4}, 0 \right)$ $\rightarrow$ $\left(\frac{3L}{4}, 0 \right)$

Question

The centre of mass of a thin rectangular plate (figure-x) with sides of length a and b , whose mass per unit area (σ) varies as $\sigma = \frac{\sigma_0 x}{ab}$ (where σ_0 is a constant), would be ____.

[28 Jan, 2025]

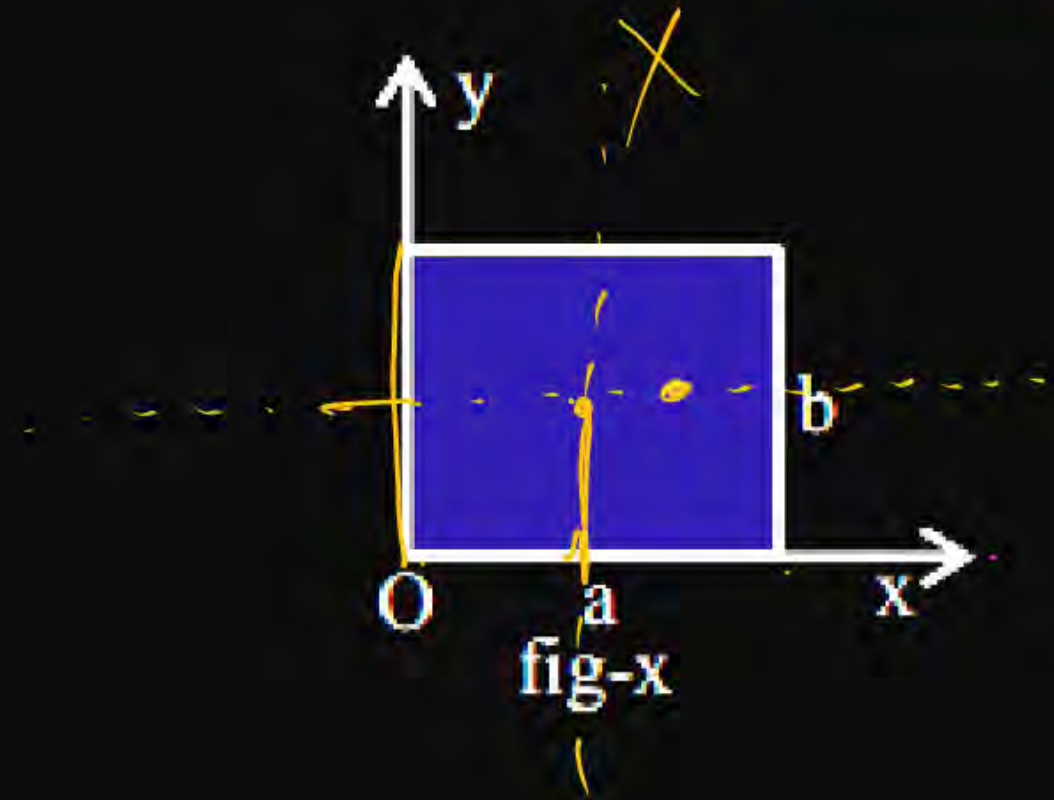
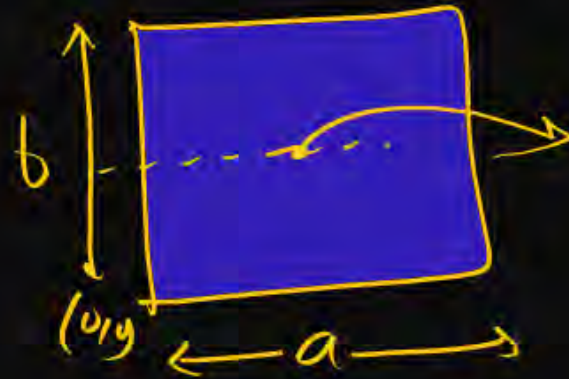
1 $\left(\frac{2}{3}a, \frac{b}{2}\right)$

2 $\left(\frac{1}{3}a, \frac{b}{2}\right)$

3 $\left(\frac{a}{2}, \frac{b}{2}\right)$

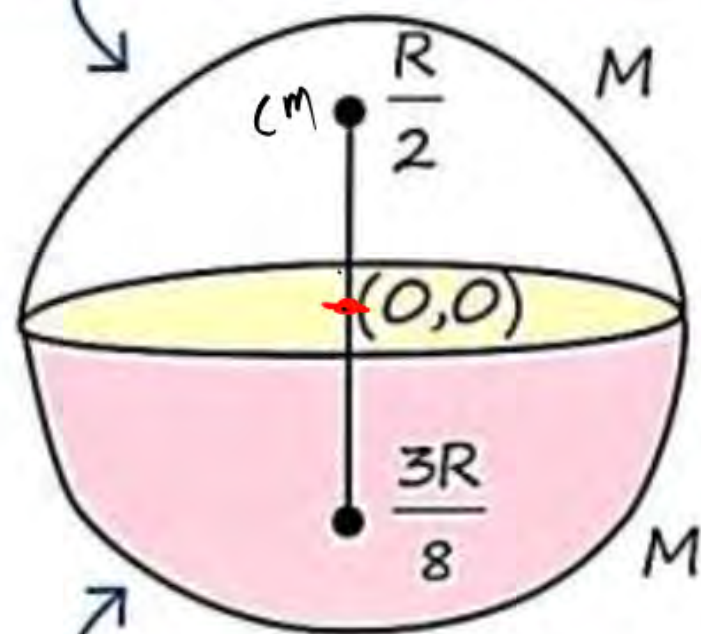
4 $\left(\frac{2}{3}a, \frac{2}{3}b\right)$

$$\sigma = \frac{\sigma_0 x}{ab}$$

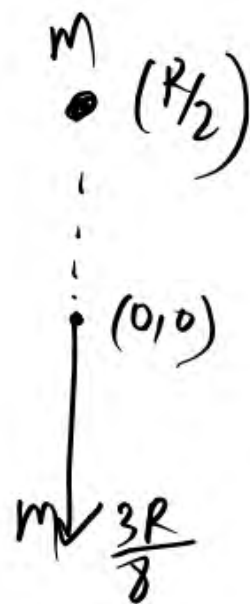


Q.9. Hollow hemisphere and solid hemisphere of same mass and radius are placed as shown. Find position of C.O.M from origin.

Sol. Hemi-hollow sphere(M)



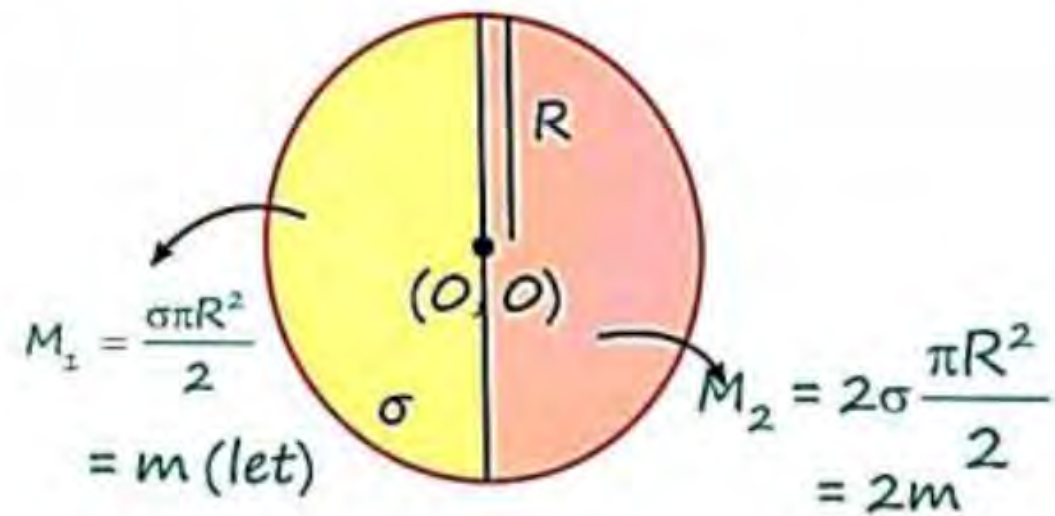
Solid-hollow sphere(M)



$$y_{cm} = \frac{m \left(\frac{R}{2}\right) \hat{j} - m \left(\frac{3R}{8}\right)}{2m}$$

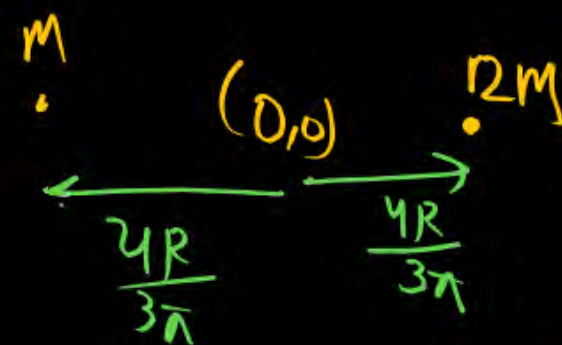
Q.10. ^{half} Two disc of same radius R of density σ and 2σ then find position of C.O.M.

Sol.



$$x_{c.m} = \frac{2m \times \frac{4R}{3\pi} - m \left(\frac{4R}{3\pi} \right)}{3m}$$

$$= \frac{4R}{9\pi}$$



$$x_{c.m} = \frac{2m \left(\frac{4R}{3\pi} \right) - m \left(\frac{4R}{3\pi} \right)}{3m} = \frac{4R}{3\pi \times 3} = \frac{4R}{9\pi}$$

Question

A uniform solid hemisphere of radius r is joined to a uniform solid right cone of base radius r . Both have same density. The centre of mass of the composite solid lies on the common face. The height (h) of the cone is JJE ADV

1 $2r$

2 $\sqrt{3}r$

3 $3r$

4 $r\sqrt{6}$

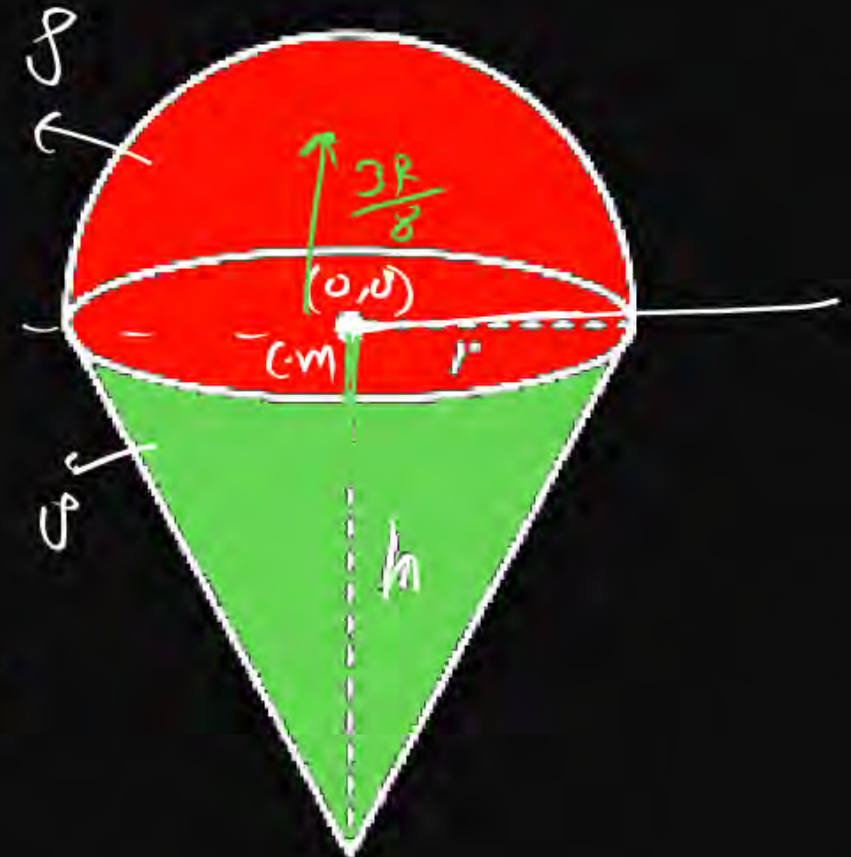
$$\bar{x}_{cm} = \frac{m_1 \bar{x}_1 + m_2 \bar{x}_2}{m_1 + m_2}$$

$$m_1 \bar{x}_1 = m_2 \bar{x}_2$$

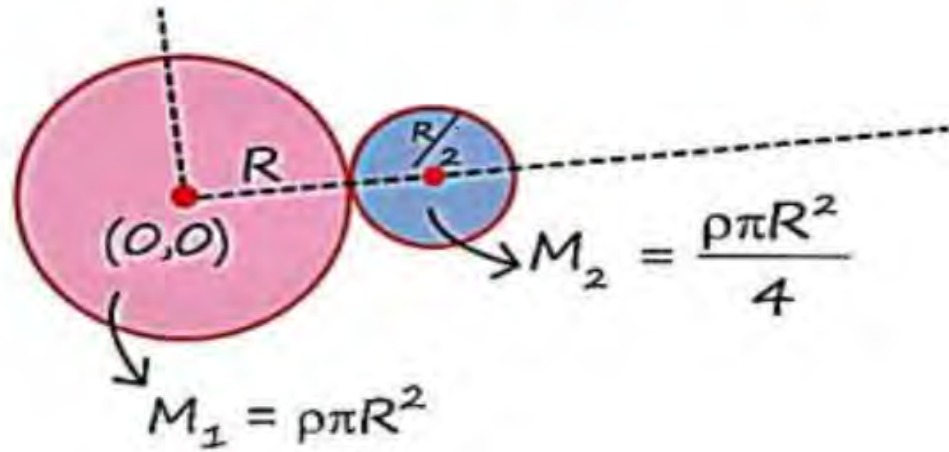
$$\cancel{\rho} \frac{1}{3} \pi r^3 \times \frac{1}{2} \times \frac{3r}{8} = \cancel{\rho} \frac{1}{3} \pi r^2 h \times \frac{h}{4}$$

$$3r^2 = h^2$$

$$h = \sqrt{3r^2} = \sqrt{3}r$$

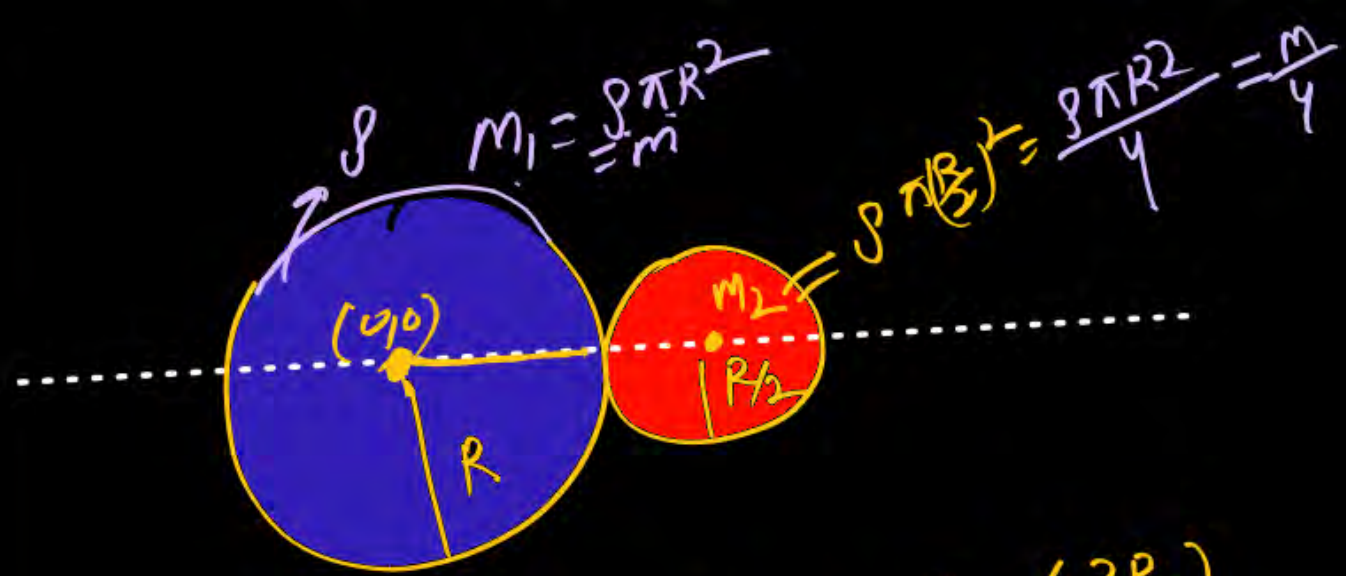


Q.11. Two disc of same density of radius R and $R/2$ as shown in figure. Find position of C.O.M of system.



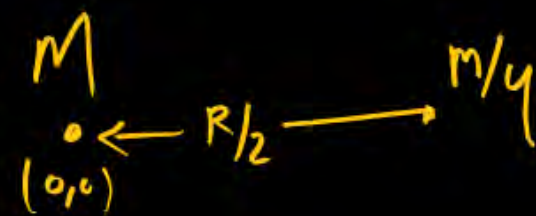
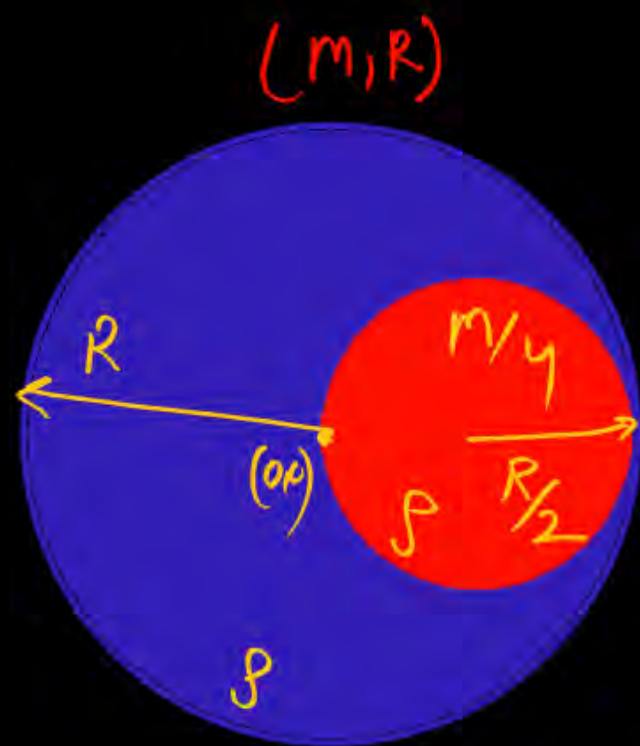
$$\text{Sol. } x_{c.m} = \frac{\rho \pi R^2 \times 0 + \frac{\rho \pi R^2}{4} \times \left(\frac{3R}{2}\right)}{\rho \pi R^2 + \frac{\rho \pi R^2}{4}}$$

$$x_{c.m} = \underline{\underline{\frac{3R}{10}}}$$



$$x_{cm} = \frac{m_1 x_0 + m_2 \left(\frac{3R}{2}\right)}{m_1 + m_2}$$

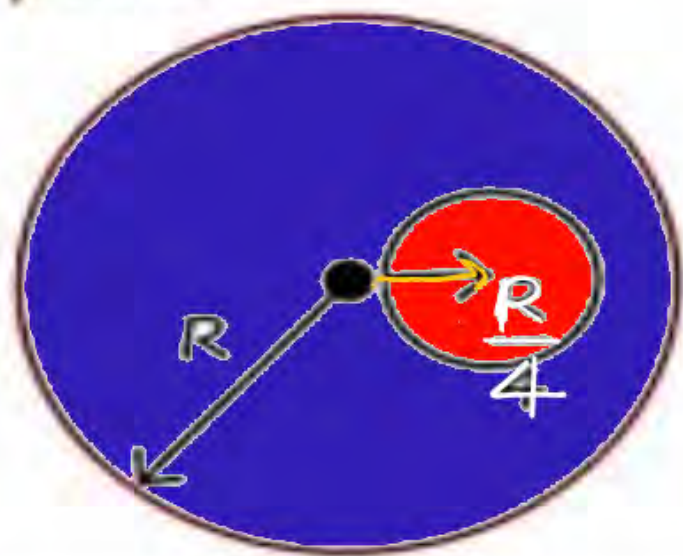
$$= \frac{\frac{m}{4} \times \frac{3R}{2}}{\frac{5m}{4}} = \frac{3R}{10}$$



$$x_{cm} = \frac{m \times 0 + \frac{m}{4} \times R/2}{m + m/4}$$

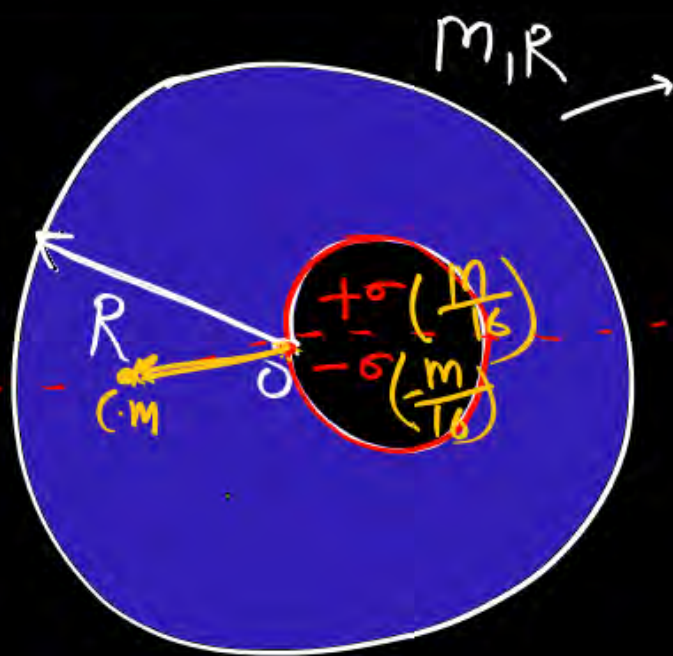
$$= \frac{\cancel{\frac{m}{4}} \times \frac{R}{2}}{5\cancel{\frac{m}{4}}} = \frac{R}{10}$$

Q.15. A disc of mass M , radius R as shown, If a disc of radius $R/4$ is cut as shown in figure then find mass of remaining part.



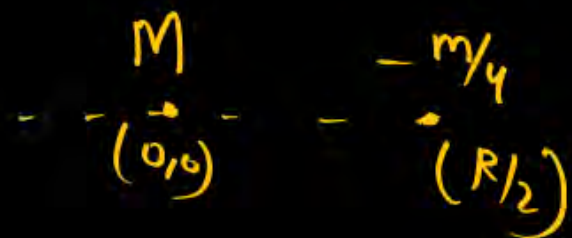
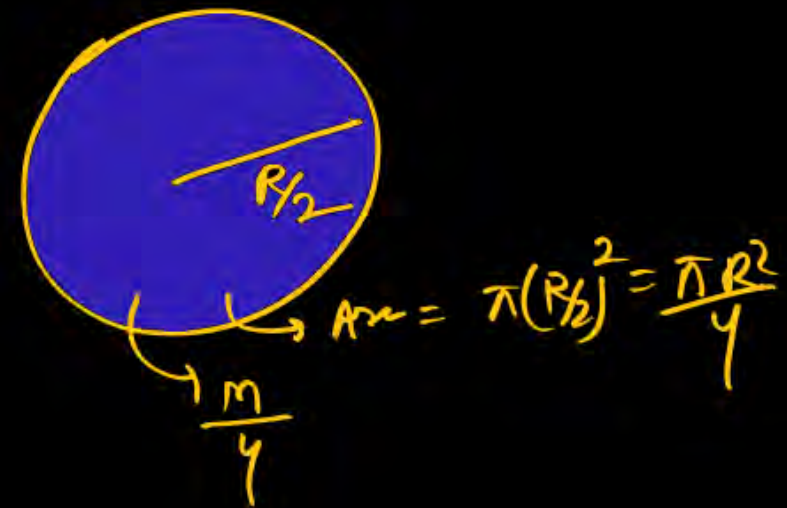
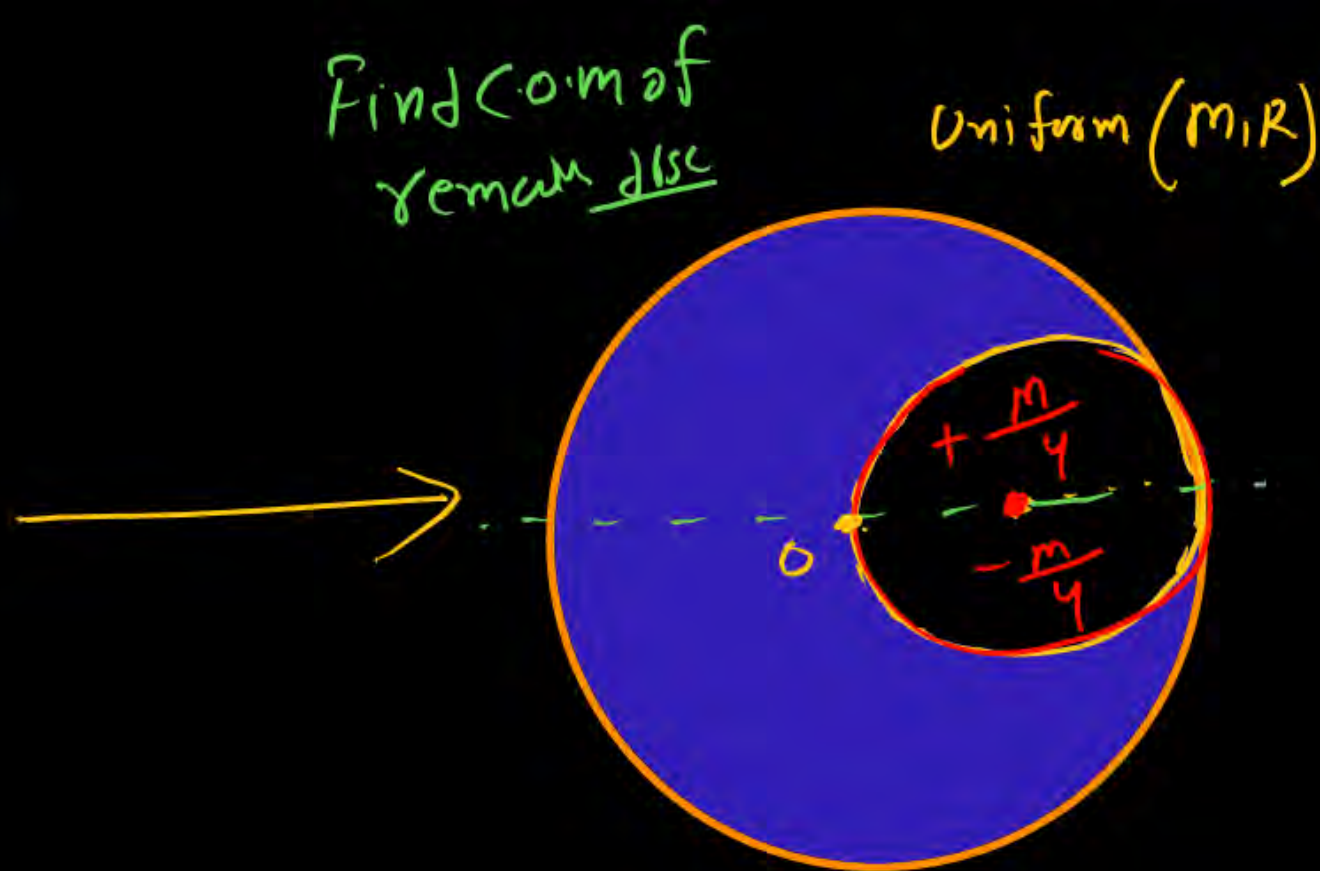
$$\text{Area} = \pi \left(\frac{R}{4}\right)^2 = \frac{\pi R^2}{16}$$

$$m' = m/16$$



$$\begin{array}{c} M \\ \bullet \\ (0,0) \end{array} \quad \begin{array}{c} -m/16 \\ \bullet \\ R/4 \end{array}$$

$$x_{cm} = \frac{m \times 0 - \frac{m}{16} \frac{R}{4}}{M - \frac{m}{16}} = \frac{-\frac{m}{16} \frac{R}{4}}{\frac{15m}{16}} = \frac{-R}{60}$$



$$x_{cm} = \frac{m \times 0 - \frac{m}{4} \times (R/2)}{m - \frac{m}{4}} =$$

$$= \frac{-\frac{m}{4} \times \frac{R}{2}}{\frac{3m}{4}} = -\frac{R}{6} \checkmark$$

MR*

+ For 2-D Mass $\propto$ Area ✓

+ For 3-D Mass $\propto$ Volume ✓

+ For 1-D Mass $\propto$ Length ✓

Question



A circular hole of radius is cut of a circular disc of radius ' a ' as shown in figure. The centroid of the remaining circular portion with respect to point ' O ' will be:

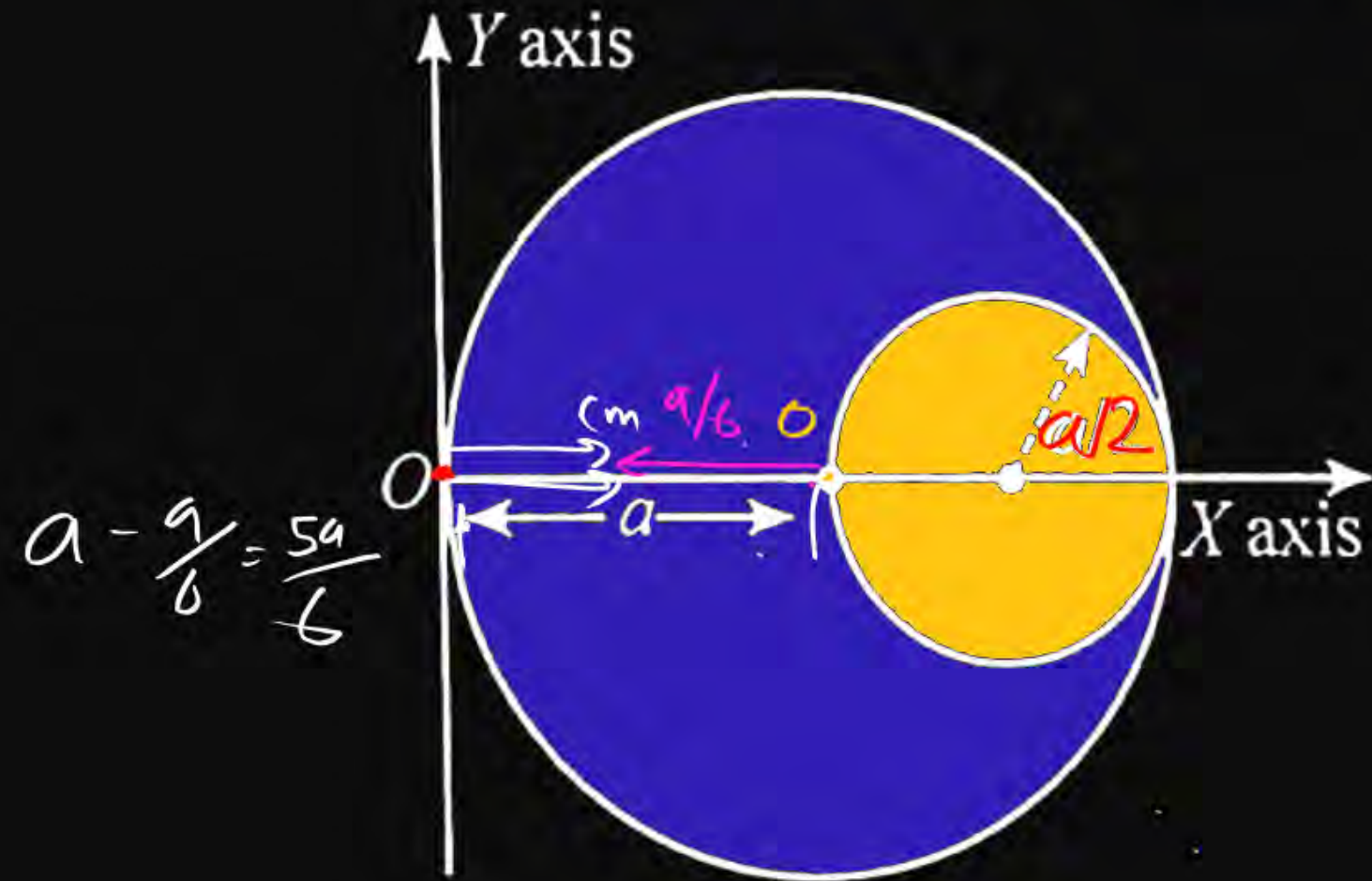
[24 Feb, 2021 (Shift-II)]

1 $\frac{5}{6}a$ ✓

2 $\frac{1}{6}a$ ✗

3 $\frac{10}{11}a$

4 $\frac{2}{3}a$

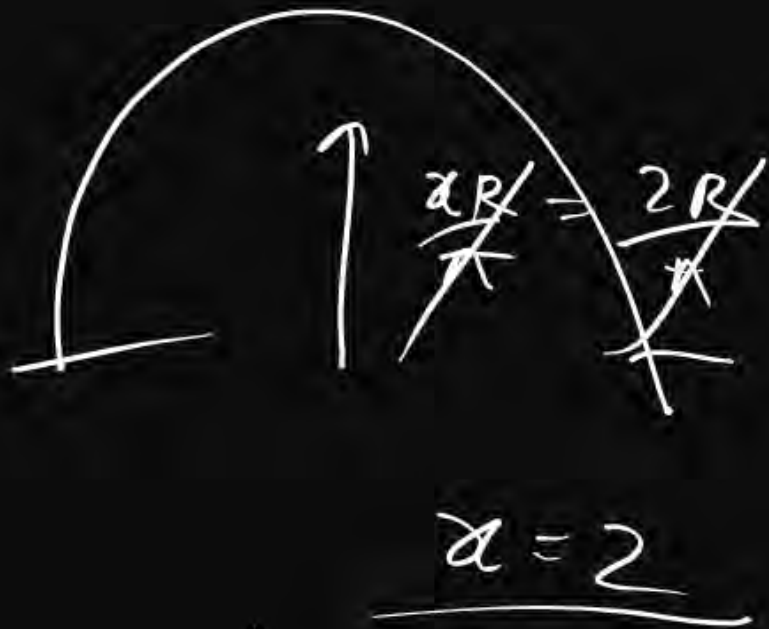


Question

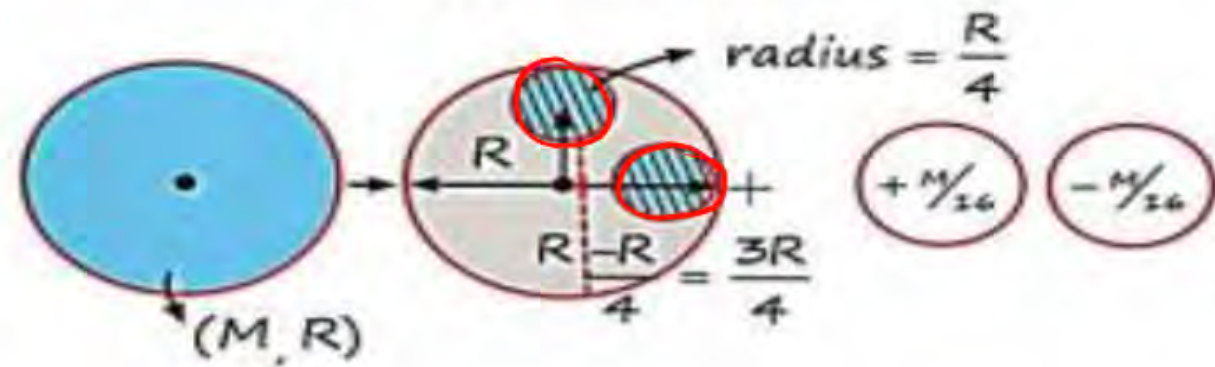


The position of the centre of mass of a uniform semi-circular wire of radius ' R ' placed in x - y plane with its centre at the origin and the line joining its ends as x -axis given by $\left(0, \frac{xR}{\pi}\right)$. Then, the value of $|x|$ is _____.

[22 July, 2021 (Shift-II)]



Q.18. From a disc of mass M and radius R , two disc of radius $R/4$ is removed as shown in figure then find position of C.O.M of remaining part:-



Sol. Mass = $\left(\frac{M}{16}\right)$

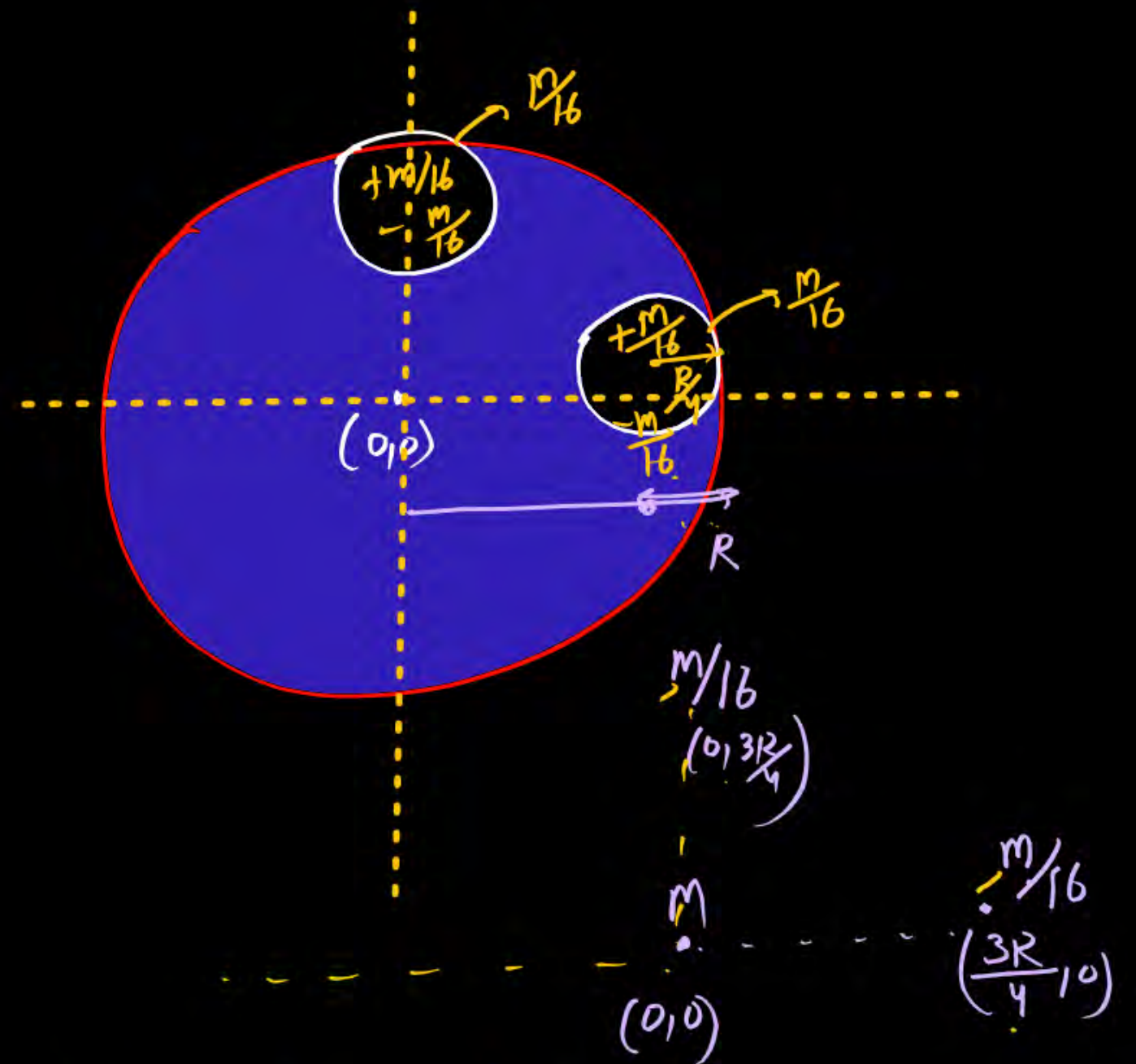
$$\text{Area} = \pi \left(\frac{R}{4}\right)^2 = \frac{\pi R^2}{16}$$

For x-coordinate:

$$x_{c.m} = \frac{M \times 0 - \frac{M}{16} \frac{3R}{4} - \frac{M}{16} \times 0}{M - \frac{M}{16} - \frac{M}{16}}$$

$$x_{c.m} = \frac{-3R}{56}$$

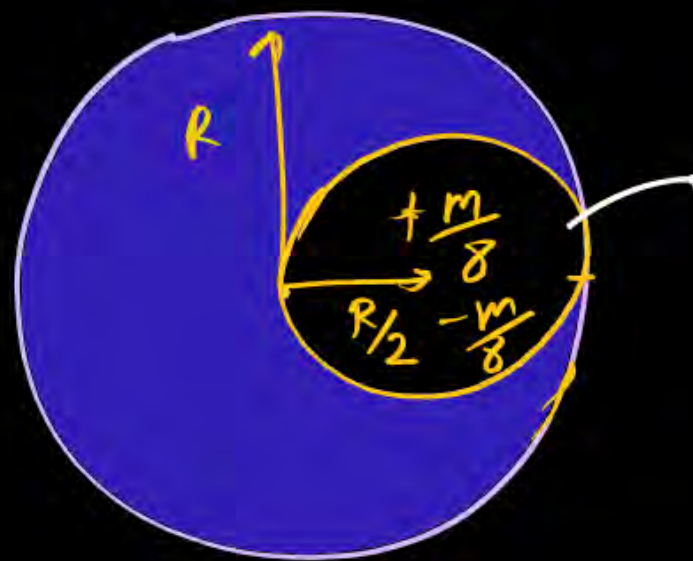
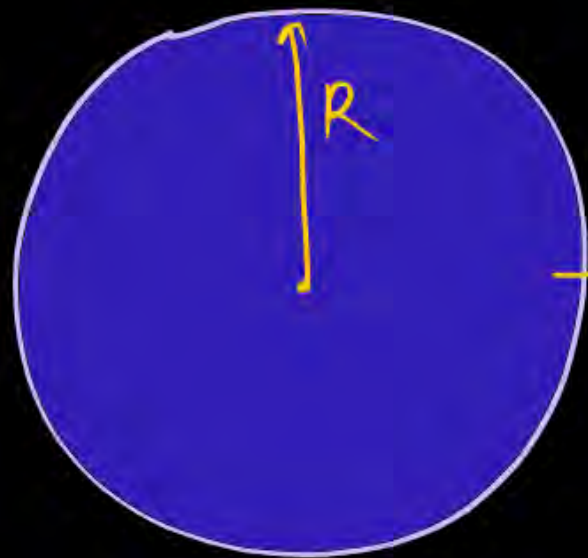
$$r_{c.m} = \left(\frac{-3R}{56} \hat{i}, \frac{-3R}{56} \hat{j}\right)$$



$$x_{c.m} = \frac{m \times 0 - \frac{m}{16} \left(\frac{3R}{4}\right) + \frac{m}{16} \times 0}{M - \frac{m}{16} - \frac{m}{16}}$$

$$= \frac{\frac{3mR}{64}}{\frac{14m}{16}} = \frac{3R}{56}$$

Solid (m, R)

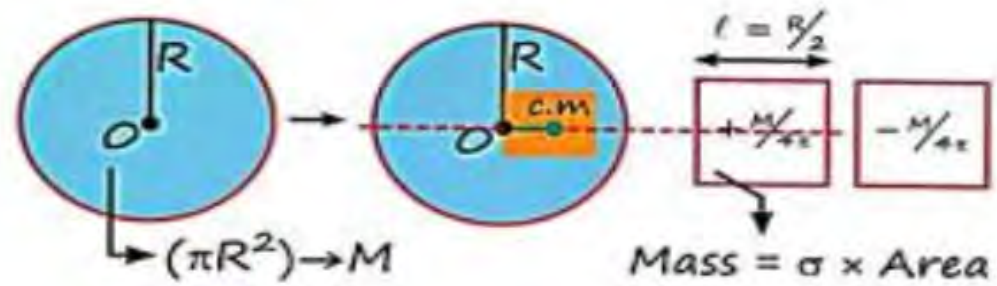


$$mass \propto \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 = \frac{4}{3}\pi \frac{R^3}{8}$$

$$\dots - \frac{m}{(0,0)} - \frac{-m/8}{(R/2)} \dots$$

$$x_{cm} = \frac{m \times 0 - \frac{m}{8} (R/2)}{\frac{7m}{8}} = \frac{R}{14}$$

Q.20. A square plate of side length $l = \frac{R}{2}$ is removed as shown in figure then find C.O.M of remaining part. Find position of C.O.M of this part: -



Sol. $(\pi R^2) \rightarrow M$

Mass per unit Area $\sigma = \frac{M}{\pi R^2}$

Mass = $\sigma \times \text{Area}$

$$= \frac{M}{\pi R^2} \times \left(\frac{R}{2}\right)^2$$

$$= \frac{M}{4\pi}$$

$$x_{c.m} = \frac{M \times 0 - \frac{M}{4\pi} \times \frac{R}{4}}{M - \frac{M}{4\pi}}$$

$$= \frac{-R}{4(4\pi - 1)}$$

MR* Box

English Alphabets:

- + Symmetrical about both axes:

I, H, O, X

- + Symmetrical about the x-axis:

~~B, C, D, E, K~~

- + Symmetrical about the y-axis:

A, M, T, U, V, W, Y

- + Not symmetrical about any axis:

F, G, J, L, N, P, Q, R, S, Z

3 Velocity of C.O.M

MR* Box

System ke momentum ke liye $V_{c.m}$ use karna hai but K.E system ke liye $V_{c.m}$ use nahi karna hai.

Q.22. Find velocity of C.O.M, momentum of system and K.E of system in given case:-

Case-1:



- $V_{c.m} = \frac{mV\hat{i} - mV\hat{i}}{2m} = 0$
- $P_{c.m} = M_{system} V_{c.m} = 0$
- $K.E = \frac{1}{2}mV_{c.m}^2$ X

$$K.E = \frac{1}{2}mV^2 + \frac{1}{2}m(-V)^2$$

✓

$K.E = mV^2$

$$K.E_{c.m} = \frac{P_{c.m}^2}{2m}$$

Case-2:



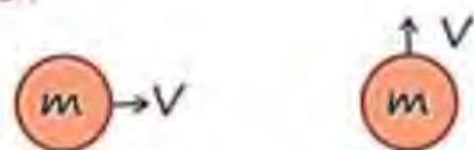
$$V_{cm} = \frac{mvi + mvi}{2m}$$

$$\vec{V}_{cm} = \frac{2mvi}{2m} = vi$$

$$\vec{P}_{cm} = \underline{2m(V)} \hat{i}$$

$$K.E = mv^2 \checkmark$$

Case-3:



$$\vec{V}_{cm} = \frac{mvi + mv\hat{j}}{2m}$$

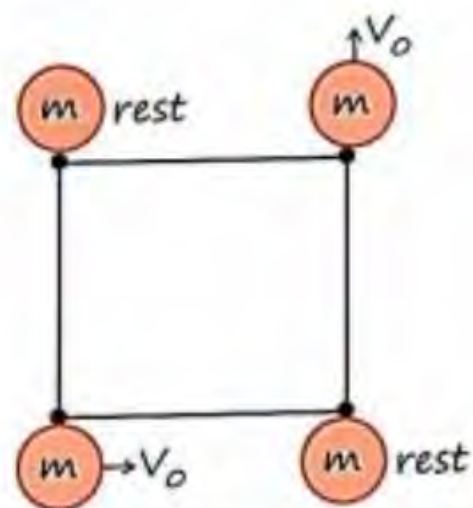
$$\vec{V}_{cm} = \frac{v}{2}\hat{i} + \frac{v}{2}\hat{j}$$

$$|\vec{V}_{cm}| = \frac{v}{2}\sqrt{2} = \frac{v}{\sqrt{2}}$$

$$|\vec{P}_{cm}| = 2m \frac{v}{\sqrt{2}} = \underline{\underline{\sqrt{2}mv}}$$

$$K.E = mv^2$$

Q.23. Find v_{cm} of the following system:



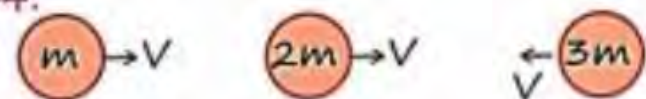
Sol. $V_{cm} = \frac{mv_0 \hat{i} + m \times 0 + mv_0 \hat{j} + m \times 0}{4m}$

$\vec{V}_{cm} = \frac{V_0}{4} \hat{i} + \frac{V_0}{4} \hat{j}$

$(K.E)_{system} = \frac{1}{2}mv_0^2 + 0 + \frac{1}{2}mv_0^2 + 0$
 $= mv_0^2$

Center of Mass

Case-4:



$$\text{Sol. } \vec{V}_{cm} = \frac{m\vec{v}_1 + 2m\vec{v}_2 - 3m\vec{v}_3}{6m}$$

$$\vec{V}_{cm} = 0 \quad \checkmark$$

$$\vec{P}_{system} = 0$$

$$\# K.E_{system} = \frac{1}{2}mv^2 + \frac{1}{2}(2m)V^2 + \frac{1}{2}(3m)V^2 \quad \checkmark$$

* Case-5:



$$\text{Sol. } \# K.E = mv^2$$

$$\# \vec{V}_{cm} = \frac{m\vec{v}_1 + m\left(\frac{V}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)}{2m} \quad \checkmark$$

$$\# P_{system} = 2m \vec{V}_{cm}$$

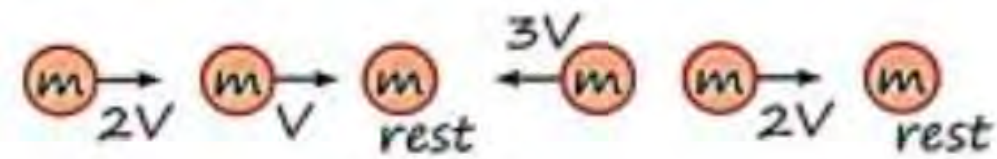
Q.24. Find V_{cm} of following system:

Sol.



$$\vec{V}_{cm} = \frac{3m \times 10 - 2m \times 5}{5m} = 4 \text{ m/s}$$

Q.25. Find V_{cm} :

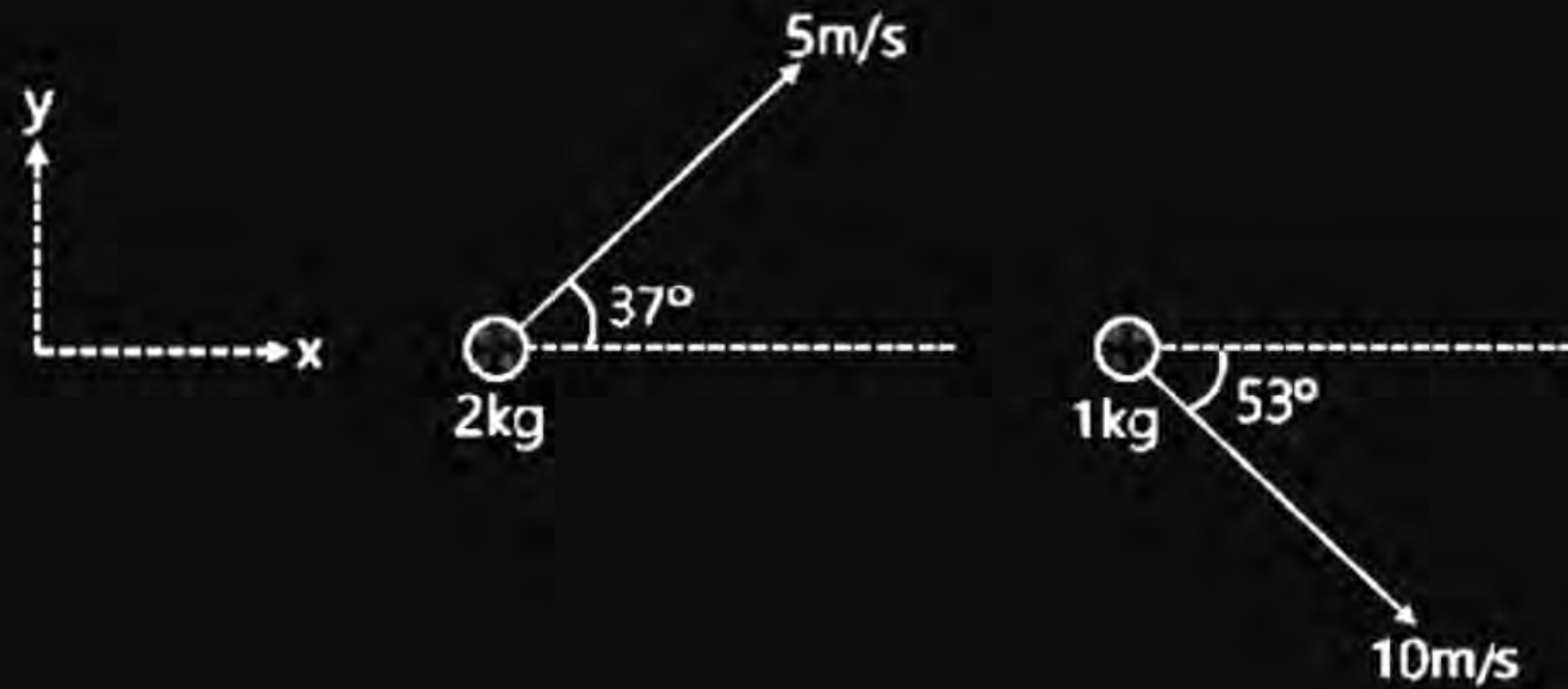


$$\text{Sol. } V_{cm} = \frac{m(2V) + mV + m \times 0 - 3mV + m(2V) + m \times 0}{6m}$$

$$V_{cm} = \frac{5mV - 3mV}{6m} = \frac{2mV}{6m} = \frac{V}{3}$$

Question

Find velocity of COM.



Question



Three particles of masses 1 kg, 2 kg and 3 kg are subjected to forces $(3\hat{i} - 2\hat{j} + 2\hat{k})\text{N}$, $(-\hat{i} + 2\hat{j} - \hat{k})\text{N}$ and $(\hat{i} + \hat{j} + \hat{k})\text{N}$ respectively. Find the magnitude of the acceleration of the CM of the system.

Question



Two particles of mass 1 kg and 3 kg are having acceleration $(\hat{i} - 2\hat{j} + 5\hat{k})$ m/s² and $(-2\hat{i} + \frac{4}{3}\hat{j} - \hat{k})$ m/s². Find out $\vec{a}_{\text{cm}}$.

THANK
YOU

MISSION

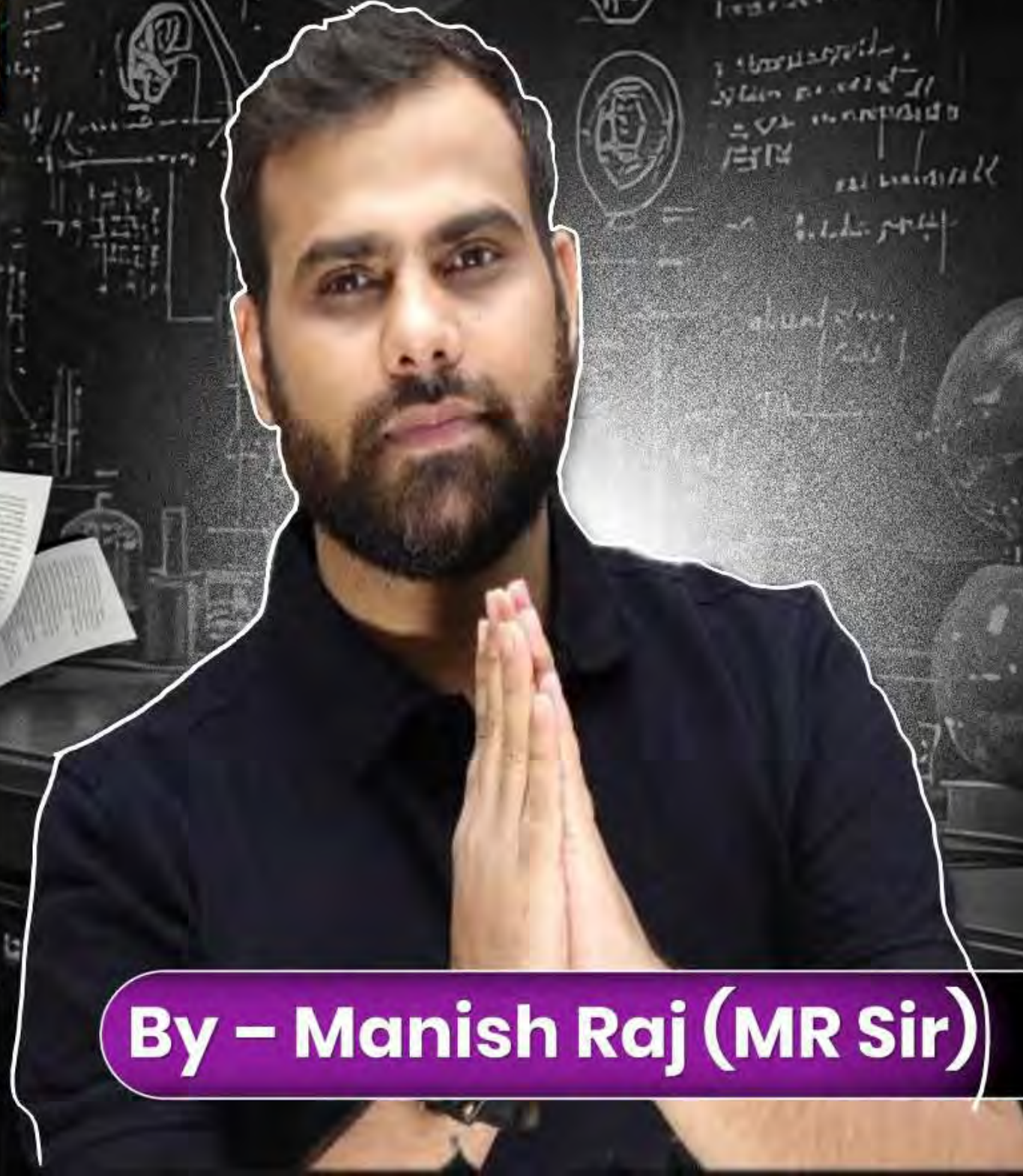
100

Centre of Mass and System of Particles

Physics

Lecture - 02

By – Manish Raj (MR Sir)



Today's Goal

*
Revision, velocity & accⁿ of COM ✓
explosion, gun-bullet, man-plank.

Numerical Types/Integer Types

30. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is
(JEE Adv. 2013)

$$P = 0.5 \text{ W} = \text{const}$$

$$t=0, u=0 \rightarrow t=5 \text{ s } v=?$$

$$P = \frac{W_{\text{net}}}{\Delta t} = \frac{K \cdot E_f - K \cdot E_i}{t}$$

$$\frac{0.5}{5} = \frac{\frac{1}{2} m v_f^2}{5}$$

$$25 = v_f^2$$

$$v_f = 5 \text{ m/s}$$

Position of centre of mass discrete system

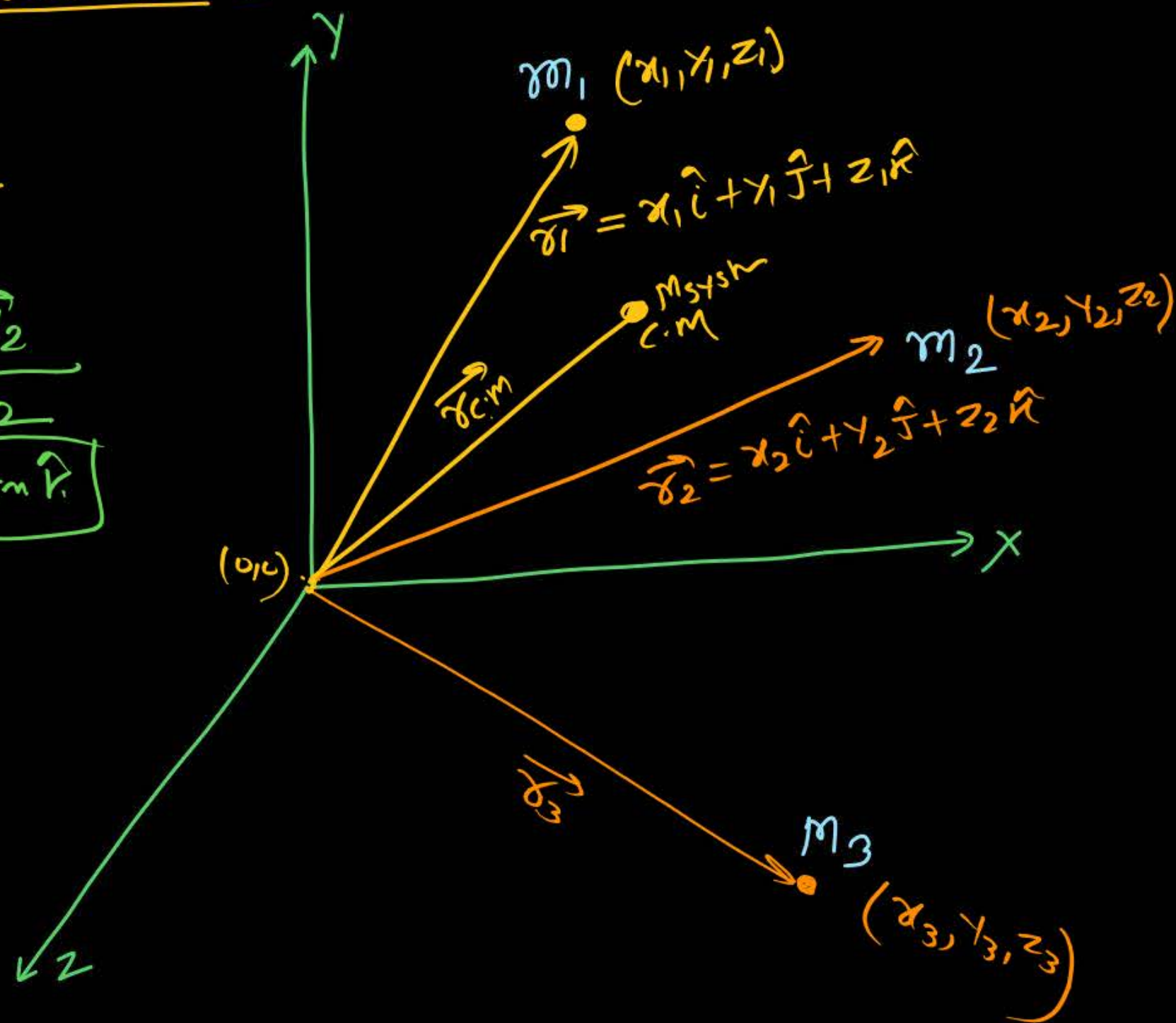
$$\# \vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3}$$

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$\vec{r}_{cm} = x_{cm} \hat{i} + y_{cm} \hat{j} + z_{cm} \hat{k}$$

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3}{m_1 + m_2 + m_3}$$

$$\vec{a}_{cm} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2}$$



C.O.M Ka concept system ka momentum & a.c.m nikalne
ke liye valid hai

but K.E of system or Moment of Inertia of
system ke calculate ke liye C.O.M
ka concept valid nahi matlb
C.O.M pe marks assume kar ke
K.E syst. & moI ka calculate nahi

Karna

~~$$K.E_{\text{system}} = \frac{1}{2} M_{\text{sys}} \times v_{\text{c.m.}}^2$$~~

✓
$$\vec{P}_{\text{system}} = M \vec{v}_{\text{c.m.}}$$
 ✓

Question



Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-3\hat{i} - 2\hat{j} + \hat{k}$ respectively. The magnitude of position vector of centre of mass of this system will be similar to the magnitude of vector:

[29 July, 2022]

~~1~~ $\hat{i} - 2\hat{j} + \hat{k}$

~~2~~ $-3\hat{i} - 2\hat{j} + \hat{k}$

~~3~~ $-2\hat{i} + 2\hat{k}$

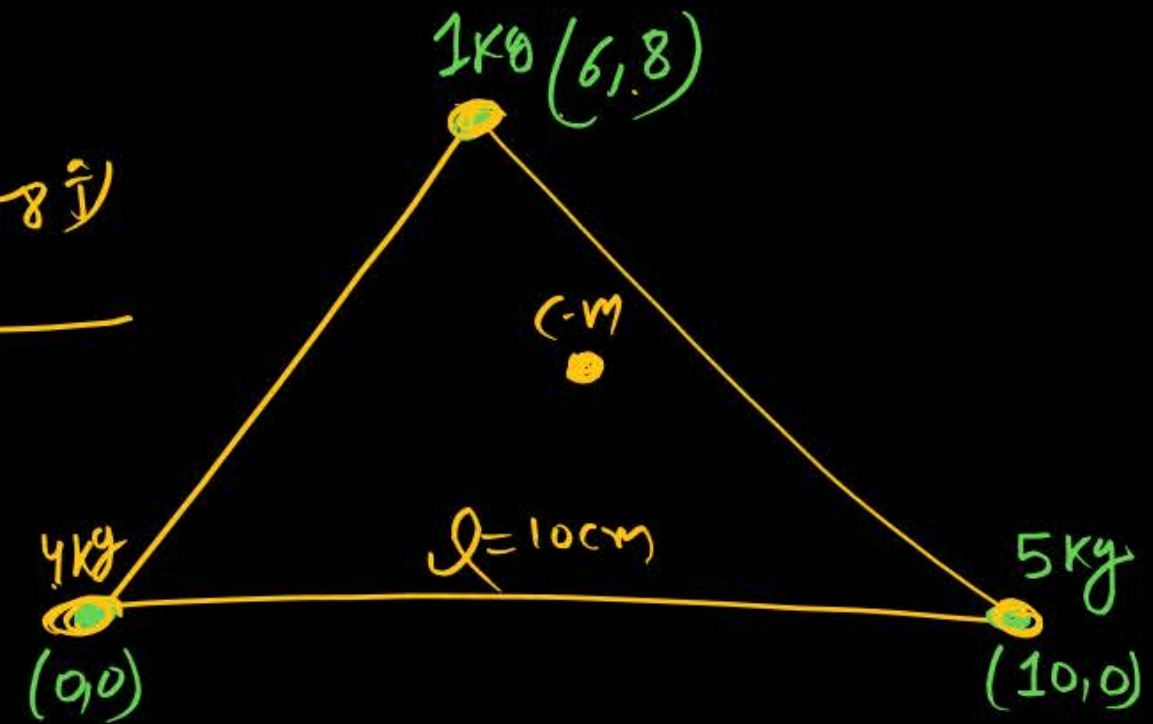
4 $-2\hat{i} - \hat{j} + \hat{k}$

$$\vec{r}_{cm} = \frac{1(\hat{i} + 2\hat{j} + \hat{k}) + 3(-3\hat{i} - 2\hat{j} + \hat{k})}{1 + 3}$$

$$= \frac{-8\hat{i}}{4} + \frac{0}{4} + \frac{\hat{k}}{1}$$

Find Position of C.O.M

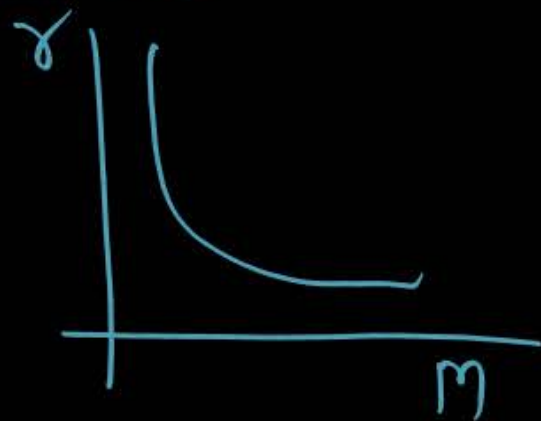
$\vec{r}_{C.M} = \frac{4 \times 0 + 5 \times 10 \hat{i} + 1(6\hat{i} + 8\hat{j})}{10}$



centre of mass of Two Particle system:-

$$m \propto \text{cost.}$$

$$x \propto \frac{1}{m}$$



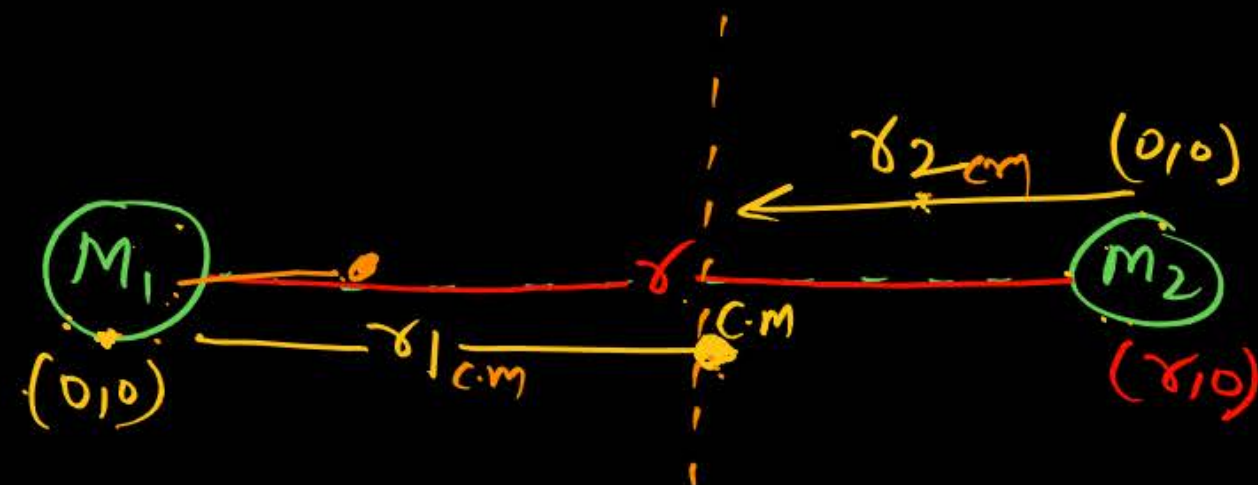
$$\frac{x_{1cm}}{x_{2cm}} = \frac{m_2}{m_1} \times \frac{m_1+m_2}{m_1+m_2}$$

$$\frac{x_{1cm}}{x_{2cm}} = \frac{m_2}{m_1}$$

$$(m_1 x_{1cm}) = (m_2 x_{2cm})$$

$$m_1 x_1 = \text{const.}$$

moment of mass



$$\vec{x}_{1cm} = \frac{m_1 \times 0 + m_2 x}{m_1 + m_2} = \frac{m_2 x}{m_1 + m_2} \hat{i}$$

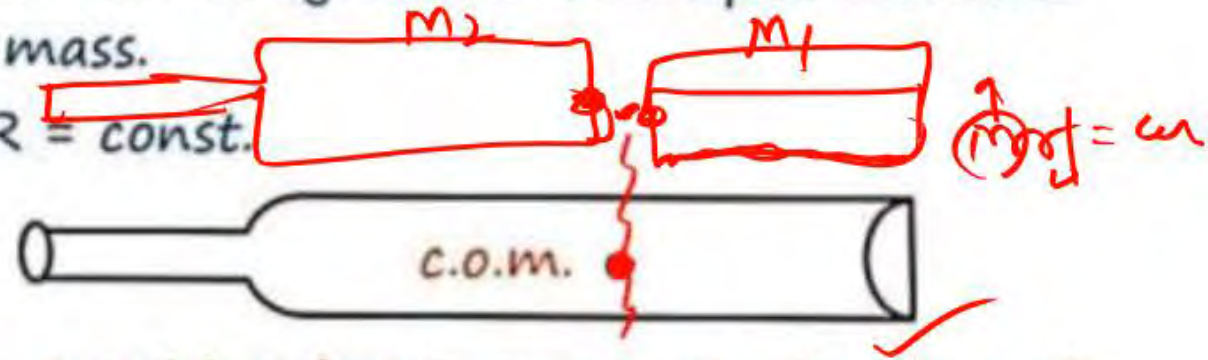
$$x_{1cm} = \frac{m_2 x}{m_1 + m_2}$$

$$x_2 = \frac{m_2 \times 0 - m_1 x}{m_1 + m_2}$$

$$x_{2cm} = \frac{m_1 x}{m_1 + m_2}$$

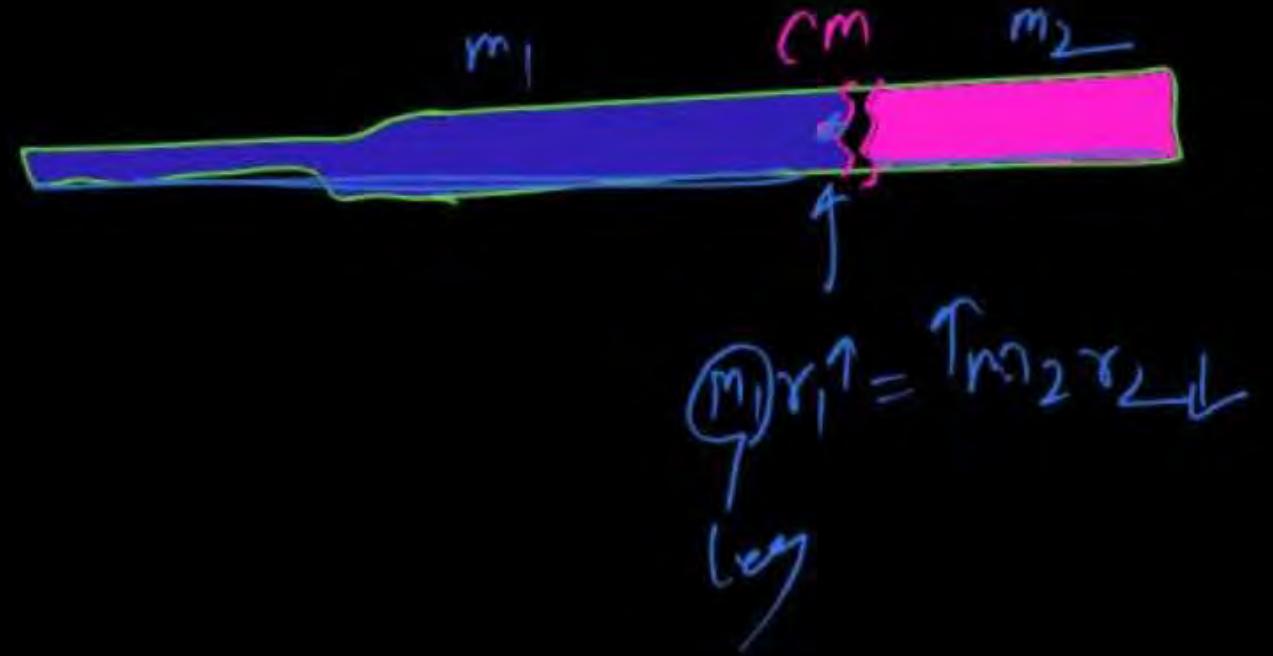
- * If we break cricket bat from c.o.m. then bottom part will have large mass, because c.o.m. divide system in two equal moment of mass.

$MR = \text{const.}$

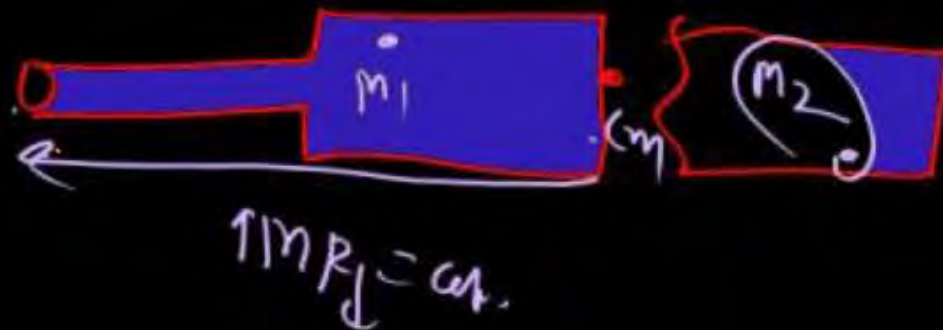


Q.6. A cricket bat is cut at the location of its centre of mass as shown. Then

- (a) The Two pieces will have the same mass
- (b) The bottom piece will have large mass
- (c) The handle piece will have larger mass
- (d) Mass of handle piece is double the mass of bottom piece



$(m_1)r_1 \uparrow = (m_2)r_2 \downarrow$



$\uparrow m_1 r_1 = \text{const.}$

Shift in C.O.M



$$\vec{\Delta r}_{C.M} = \frac{m_1 \vec{\Delta r}_1 + m_2 \vec{\Delta r}_2}{m_1 + m_2} \quad \checkmark$$

↓
If C.O.M does not change position

$$\vec{\Delta r}_{C.M} = \frac{m_1 \vec{\Delta r}_1 + m_2 \vec{\Delta r}_2}{m_1 + m_2}$$

$$\underbrace{m_1 \vec{\Delta r}_1} = - \underbrace{m_2 \vec{\Delta r}_2}$$

Linear mass density

1-D



$$\lambda = \frac{dm}{dL}$$

$$\int dm = \int \lambda dL$$

$$\int dm = \int \lambda dL$$

$$M = \int_0^L \lambda dL$$

Variable

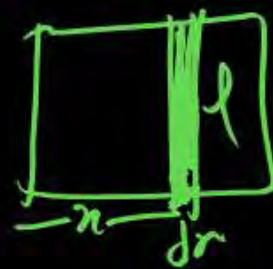
$$M = \lambda \int_0^L dL$$

$$M = \lambda L$$

$$M = \lambda L$$

$M \propto L$

Areal mass density



$$\sigma = \frac{dm}{dA}$$

$$\int dm = \int \sigma dA$$

$$Mass = \int \sigma dA$$

$$\sigma = \text{const}$$

$$Mass \propto Area$$



Volume mass density

$$\rho = \frac{dm}{dV}$$

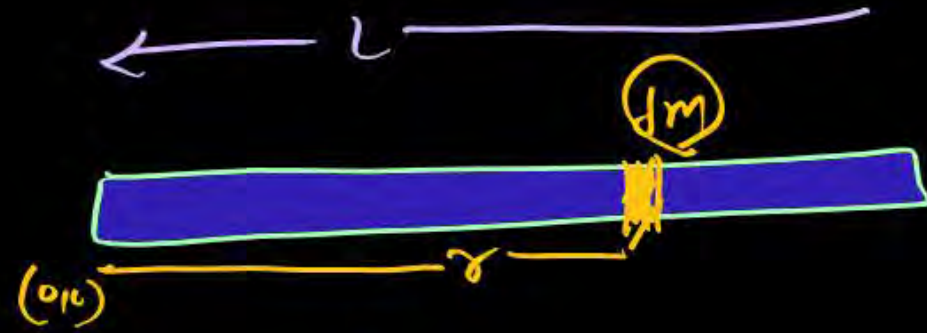
$$dm = \rho dV$$

$$\rho = \text{const}$$

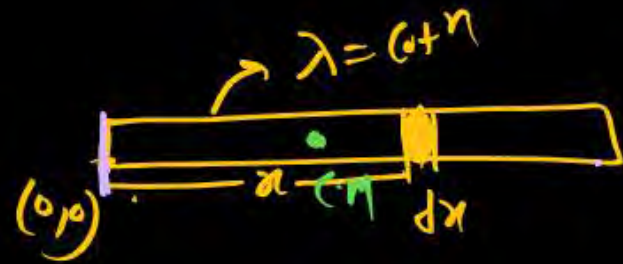
$$Mass = \rho \text{ volume}$$

$$Mass \propto \text{volume}$$

Centre of mass of continuous mass system.



$$\vec{r}_{c.m} = \frac{\int x dm}{\int dm}$$



$$dm = \lambda dx$$

$$\vec{r}_{c.m} = \frac{\int x dm}{\int dm} = \frac{\int x \lambda dx}{\int \lambda dx} = \frac{\left(\frac{x^2}{2}\right)_0^L}{\left(\frac{x}{n+1}\right)_0^L} = \frac{\frac{L^2}{2} - 0}{\frac{L}{n+1} - 0} = \frac{L}{2}$$

MR*

$$\lambda \propto Cx^n \quad x_{cm} = \frac{L}{2}$$

$$\lambda \propto x \quad \frac{L}{2} < x < L$$

$$\lambda \propto x^2 \quad \frac{L}{2} < x < L$$

* $\lambda \propto \frac{1}{x} \quad x_{cm} < \frac{L}{2}$

Question

If linear density of a rod of length 3m varies as $\lambda = 2 + x$, then the position of the center of gravity of the rod is

1 $\frac{7}{3} \text{ m} = \underline{2.3 \text{ m}}$

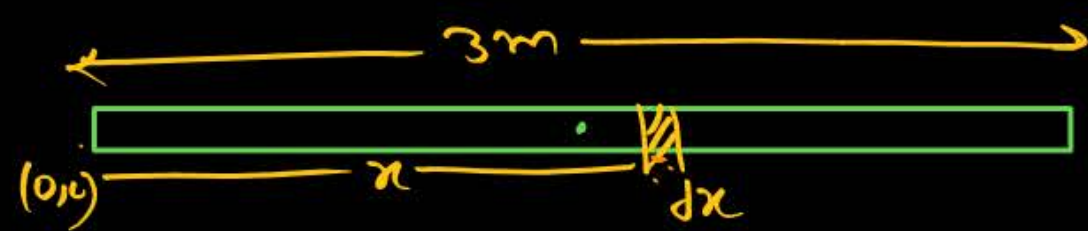
2 $\frac{12}{7} \text{ m} = 1.7 \text{ m}$

3 $\frac{10}{7} \text{ m} = 1.4$

4 $\frac{9}{7} \text{ m} = 1.1$



$$\lambda = 2 + x$$



$$dm = \lambda dx$$

$$dm = (2 + x) dx$$

$$\vec{x}_{cm} = \frac{\int x dm}{\int dm} = \frac{\int x (2 + x) dx}{\int (2 + x) dx} = \frac{\int (2x) dx + \int (x^2) dx}{\int 2 dx + \int x dx} = \frac{\left[2 \frac{x^2}{2} + \frac{x^3}{3} \right]_0^3}{\left[2(x) + \frac{x^2}{2} \right]_0^3} = \frac{x \left(x + \frac{x^2}{3} \right)_0^3}{x \left[2 + \frac{x}{2} \right]_0^3} = \frac{3 + \frac{9}{3}}{2 + \frac{3}{2}} = \frac{6}{\frac{7}{2}} = \frac{12}{7} \text{ m}$$

Question



A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$ where, a and b are constants and $0 \leq x \leq L$. The value of x for the centre of mass of the rod is:
[9 Jan, 2020 (Shift-II)]

1 $\frac{4}{3} \left(\frac{a + b}{2a + 3b} \right) L$ ✗

2 $\frac{3}{4} \left(\frac{2a + b}{3a + b} \right) L = \frac{L}{2}$ ✓

3 $\frac{3}{2} \left(\frac{2a + b}{3a + b} \right) L$

4 $\frac{3}{2} \left(\frac{a + b}{2a + b} \right) L$

$$x_{cm} = \frac{\int x \rho dx}{\int \rho dx}$$



$$\rho = a + b \left(\frac{x}{L} \right)^2$$

$$\rho = a + \frac{bx^2}{L^2}$$

$$\text{MR}^+ \quad b=0$$

$$\rho = a \rightarrow \text{or}$$

$$x = \frac{L}{2}$$

Question



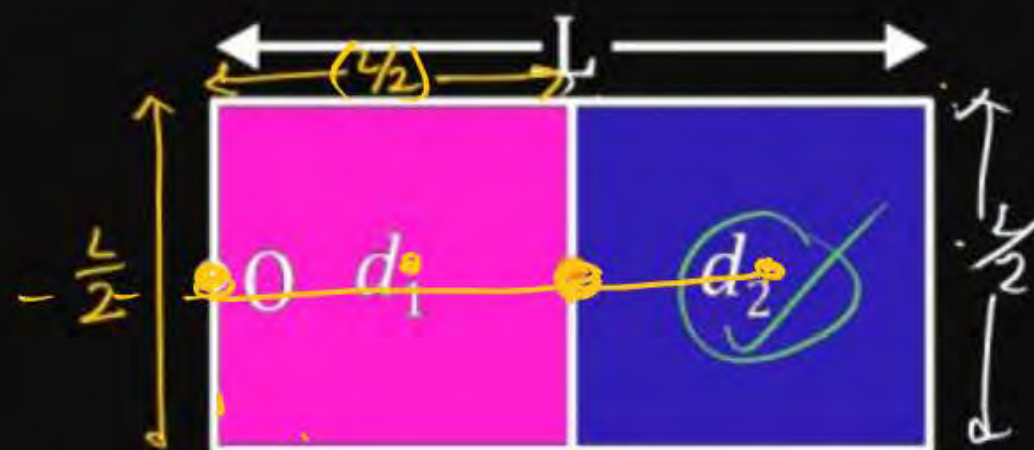
Shows a rectangular plane of length L , the half of which is made of material of density d_1 and another half of density d_2 . The distance of centre of mass from the origin O is

1 $\frac{d_1 + 2d_2}{2(d_1 + d_2)} L = \frac{3L}{4}$
 $\frac{mR^*}{\text{if } d_1 = d_2 = 1}$
 $x_m = \frac{L}{2}$

2 $\frac{(d_1 + 3d_2)L}{4(d_1 + d_2)} = \frac{4L}{8} = \frac{L}{2}$
 $\frac{mR^*}{\text{if } d_2 = 0}$
 $x_m = \frac{L}{4}$

3 $\frac{(d_1 + 3d_2)L}{2(d_1 + d_2)}$
 $\frac{mR^*}{\text{if } d_2 = 0}$
 $x_m = \frac{L}{4}$

4 $\frac{(3d_1 + d_2)L}{4(d_1 + d_2)} = \frac{4L}{8} = \frac{L}{2}$



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{d_1 \frac{L^2}{4} \times \frac{L}{4} + d_2 \frac{L^2}{4} \times \left(\frac{3L}{4}\right)}{d_1 \frac{L^2}{4} + d_2 \frac{L^2}{4}}$$

$$= \frac{(d_1 + 3d_2) \frac{L}{4}}{d_1 + d_2}$$

$(0,0)$ $\frac{m_1 = d_1 \frac{L^2}{4}}{(\frac{L}{4}, \frac{L}{4})}$ $\frac{m_2 = d_2 \frac{L^2}{4}}{(\frac{3L}{4}, \frac{L}{4})}$

Question

The centre of mass of a thin rectangular plate (figure-x) with sides of length a and b , whose mass per unit area (σ) varies as $\sigma = \frac{\sigma_0 x}{ab}$ (where σ_0 is a constant), would be ____.

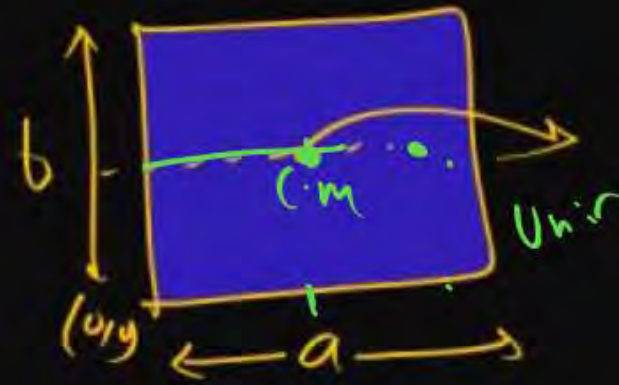
[28 Jan, 2025]

1 $\left(\frac{2}{3}a, \frac{b}{2}\right)$

2 $\left(\frac{1}{3}a, \frac{b}{2}\right)$

3 $\left(\frac{a}{2}, \frac{b}{2}\right)$

4 $\left(\frac{2}{3}a, \frac{2}{3}b\right)$



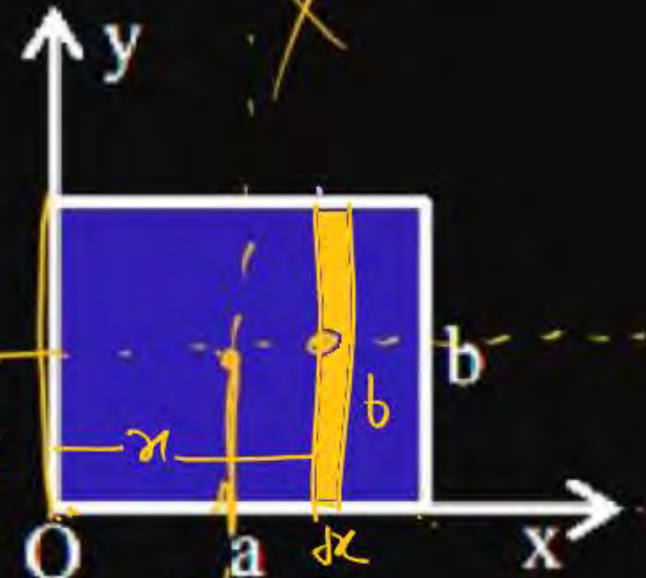
$$x_{cm} = \frac{\int x dm}{\int dm}$$

$$= \frac{\int_0^a x \cdot \sigma \cdot b dx}{\int_0^a \sigma \cdot b dx}$$

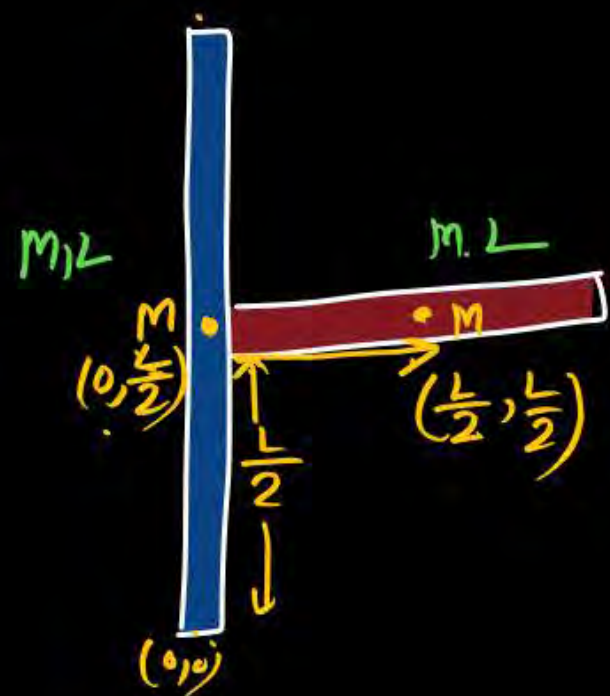
$$= \frac{\int_0^a x^2 dx}{\int_0^a x dx} = \frac{\frac{x^3}{3} \Big|_0^a}{\frac{x^2}{2} \Big|_0^a} = \frac{\frac{a^3}{3}}{\frac{a^2}{2}} = \frac{2a}{3}$$

$$dm = \frac{\sigma_0 x}{ab} \cdot b dx = \frac{\sigma_0 x}{a} dx$$

$$= \frac{\sigma_0 x}{a} dx$$



Two Identical Rod (m, L), find position of c.o.m

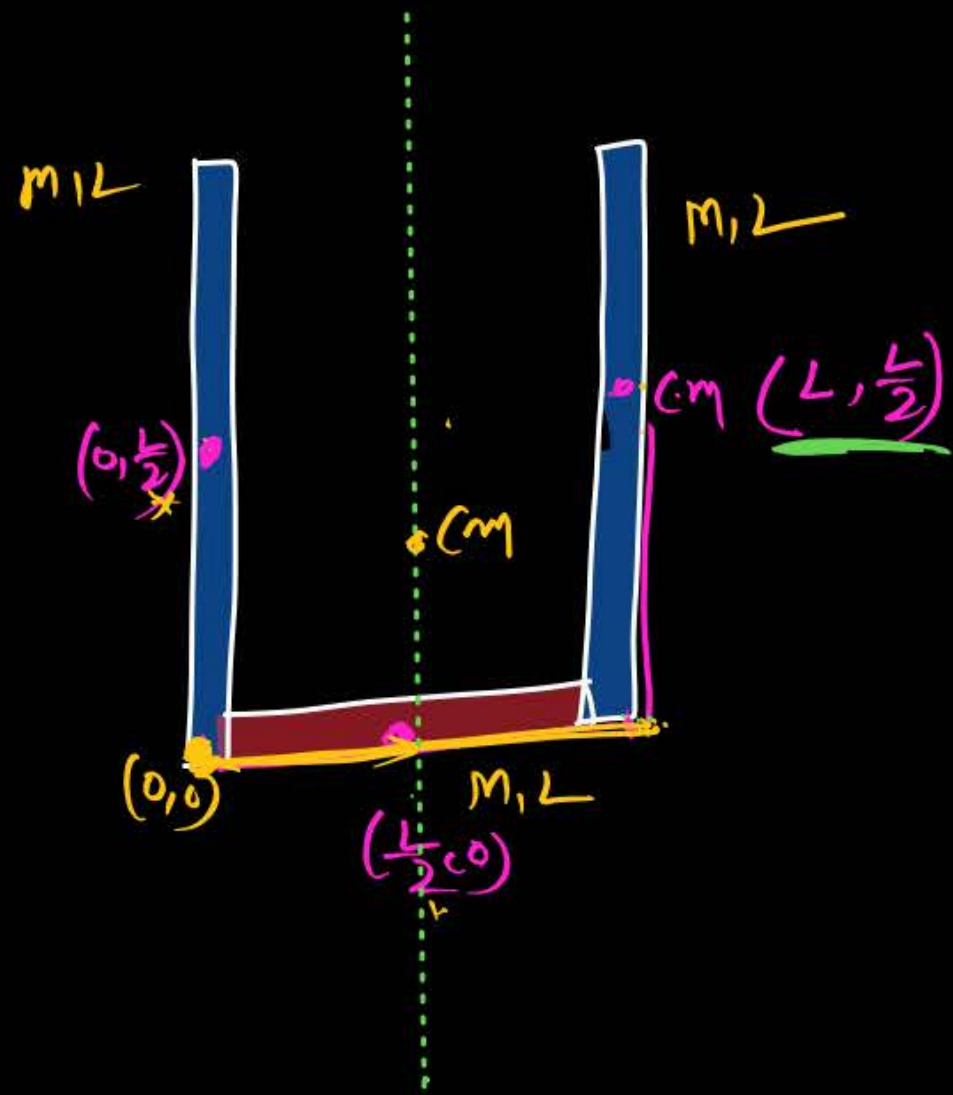


MR* Box -

collection of continuous
body $\rightarrow$ Sab ko hta
ke point man lo
uske c.o.m. pe

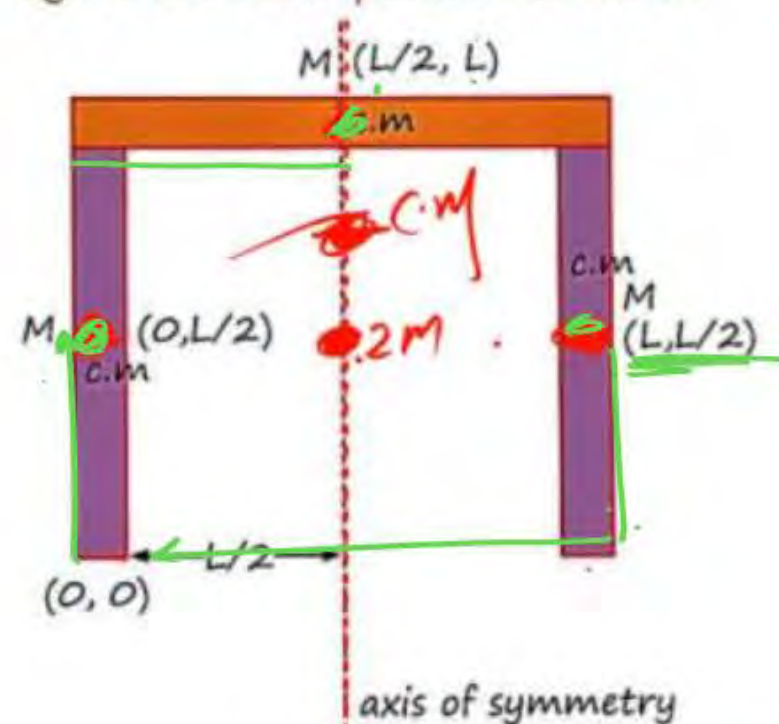
$$r_{cm} = \frac{m(0\hat{i} + \frac{L}{2}\hat{j}) + m(\frac{L}{2}\hat{i} + \frac{L}{2}\hat{j})}{2m}$$

Three Identical Rod (m, L), Find position of C.O.M.



$$\vec{r}_{cm} = \frac{m\left(\frac{L}{2}\hat{j}\right) + m\left(\frac{L}{2}\hat{i}\right) + m\left(L\hat{i} + \frac{L}{2}\hat{j}\right)}{3m}$$

Q.8. Three identical rod (M, L) as shown in figure then find position of C.O.M.



$$\text{Sol. } \vec{r}_{cm} = \frac{M \frac{L}{2} \hat{j} + M \frac{L}{2} \hat{i} + mL \hat{j} + ML \hat{i} + \frac{ML \hat{j}}{2}}{3M}$$

$$= \frac{L}{2} \hat{i} + \frac{2L}{3} \hat{j}$$

MR* Box

+ Rod ko hatakar point object maan lo Rod ke CoM pe.

$$M \left(\frac{L}{2}, L \right)$$

$$M \left(0, \frac{L}{2} \right)$$

$$M \left(L, \frac{L}{2} \right)$$

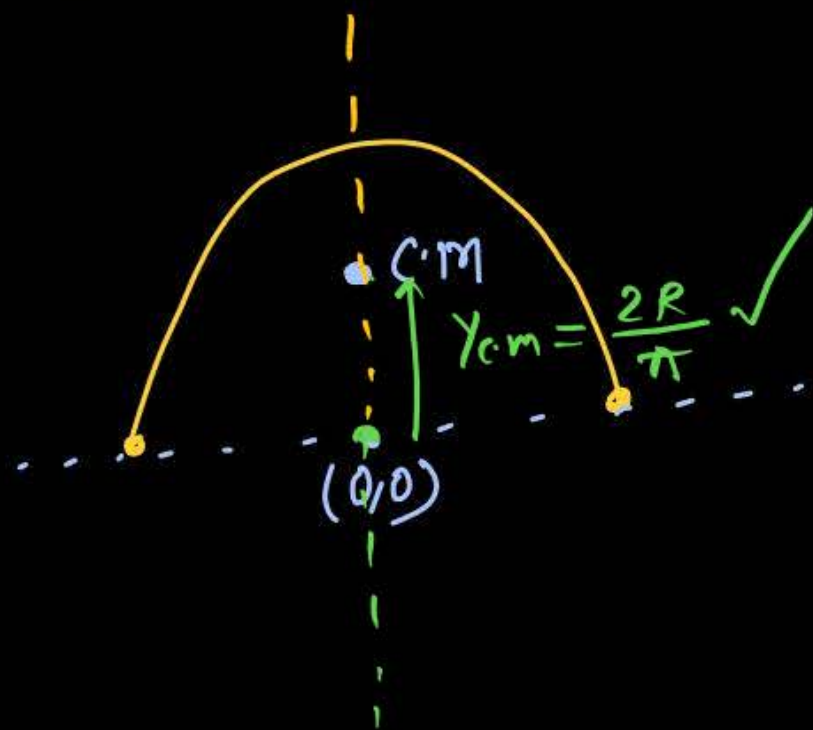
$$(0,0)$$

$$\vec{r}_{cm} = \frac{M \left(\frac{L}{2} \right) \hat{i} + M \left(\frac{L}{2} \hat{i} + L \hat{j} \right) + M \left(L \hat{i} + \frac{L}{2} \hat{j} \right)}{3M}$$

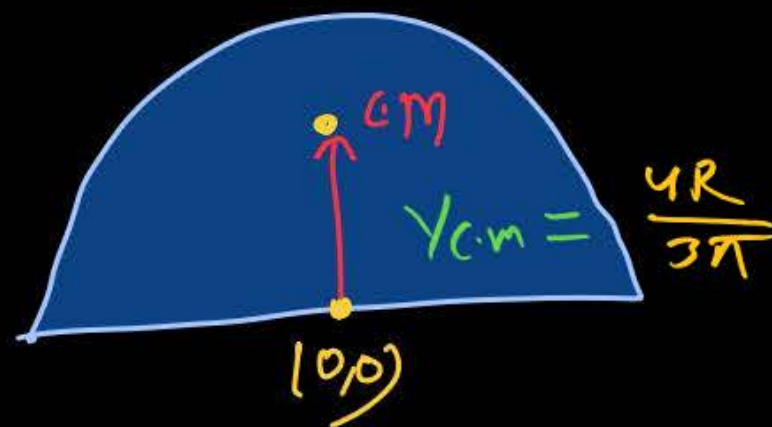
$$= \frac{\frac{3L}{2} \hat{i} + 2L \hat{j}}{3}$$

$$= \frac{L}{2} \hat{i} + \frac{2L}{3} \hat{j}$$

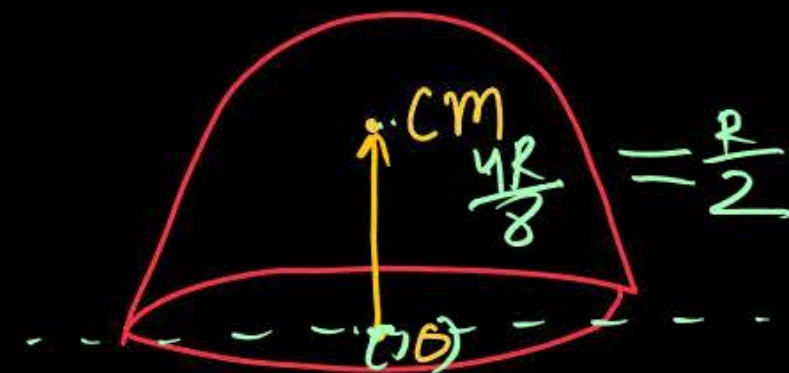
Half Ring



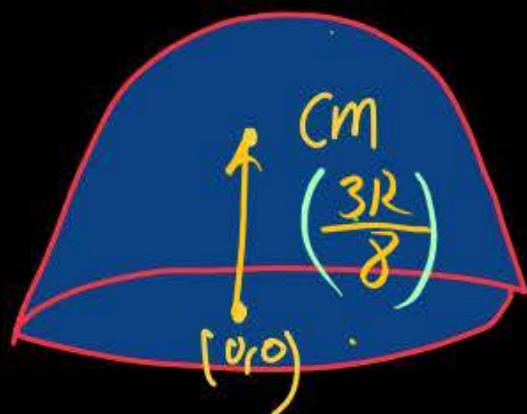
Half disc



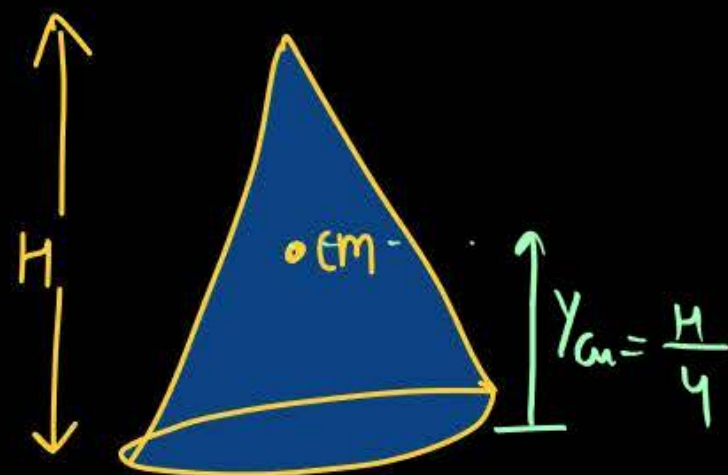
Hollow sphere



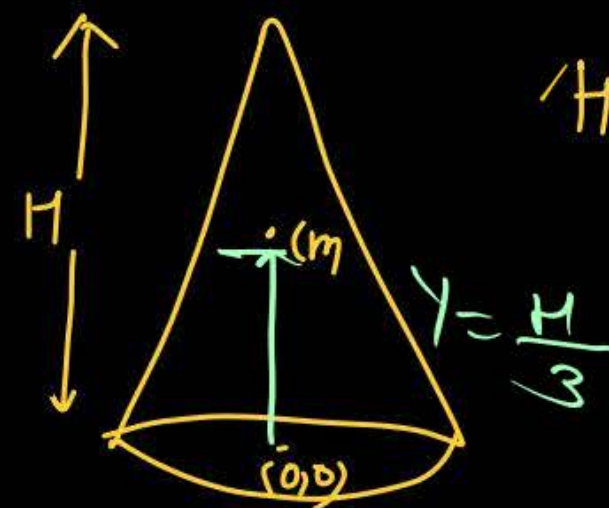
Solid - sphere



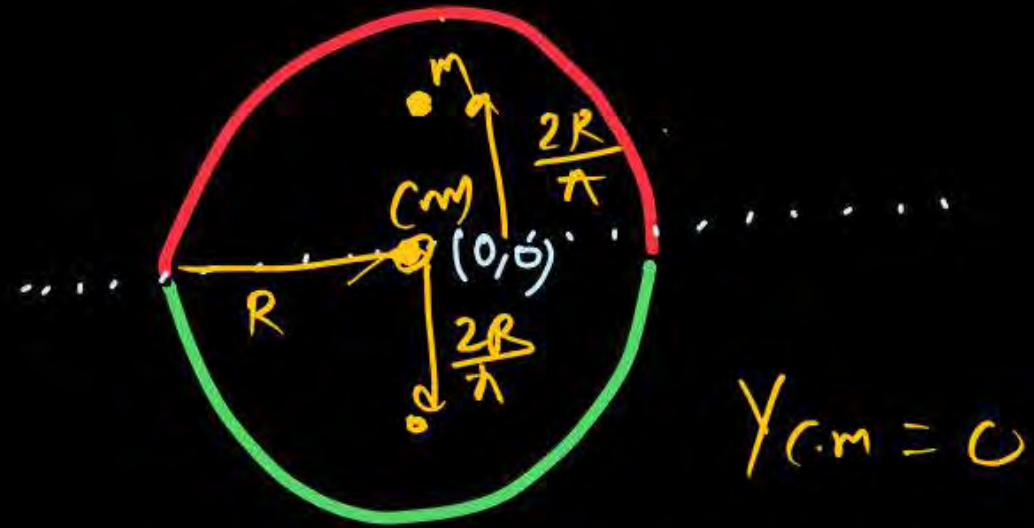
solid



Hollow cone



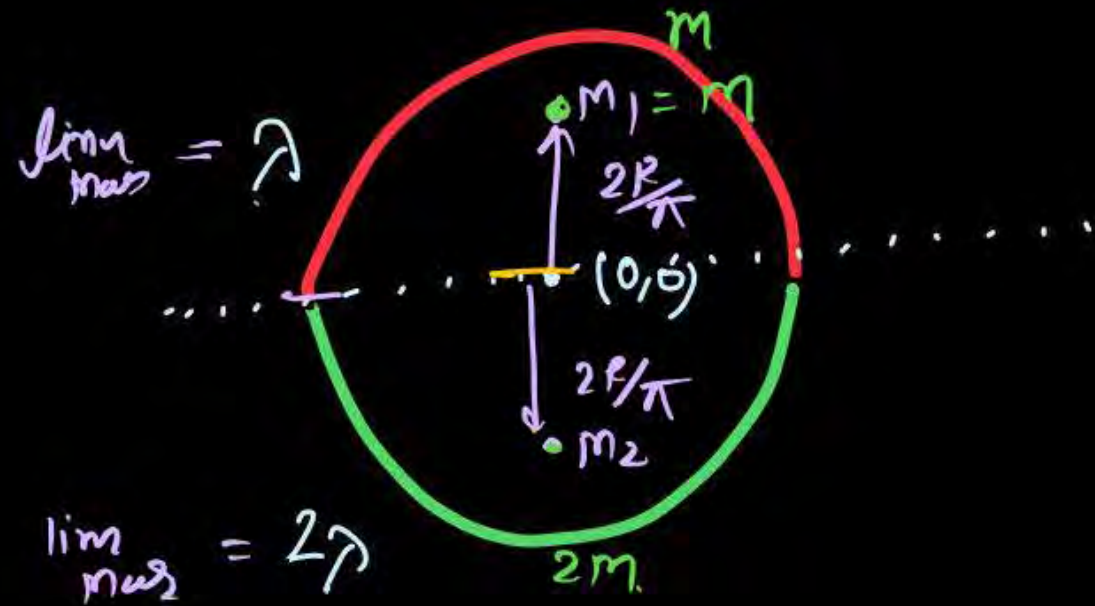
Two half Ring of same mass then find C.O.M of system:—



$$Y_{C.M} = 0$$

$$Y_{C.M} = \frac{m(\frac{2R}{\pi}) - m(\frac{2R}{\pi})}{2m} = 0$$

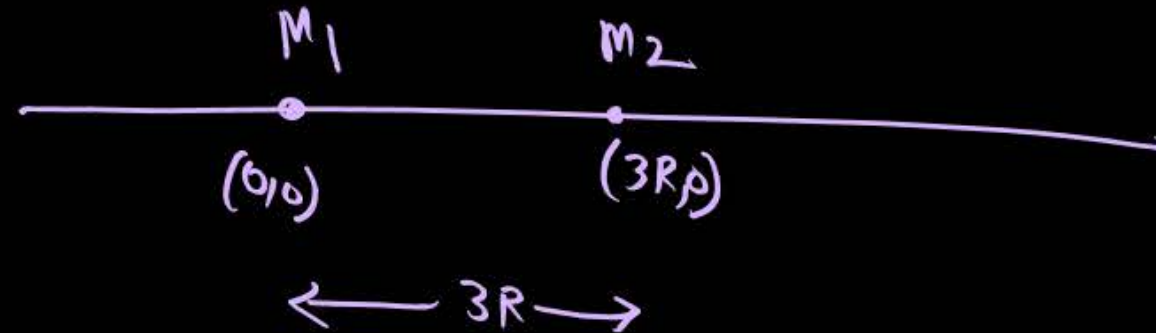
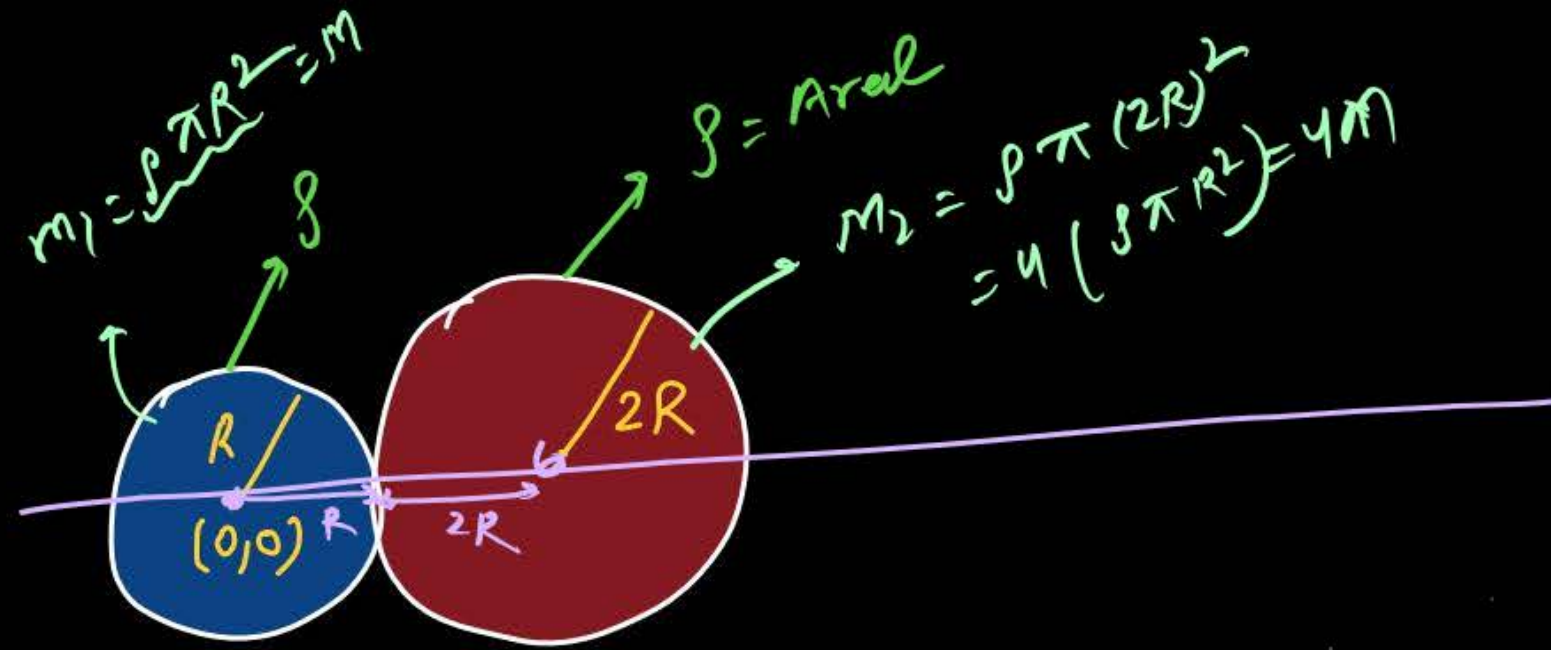
Two half Ring of same Radius then find C.O.M of System:—



$$Y_{C.M} = \frac{m_1 \frac{2R}{\pi} - m_2 \left(\frac{2R}{\pi} \right)}{m_1 + m_2}$$

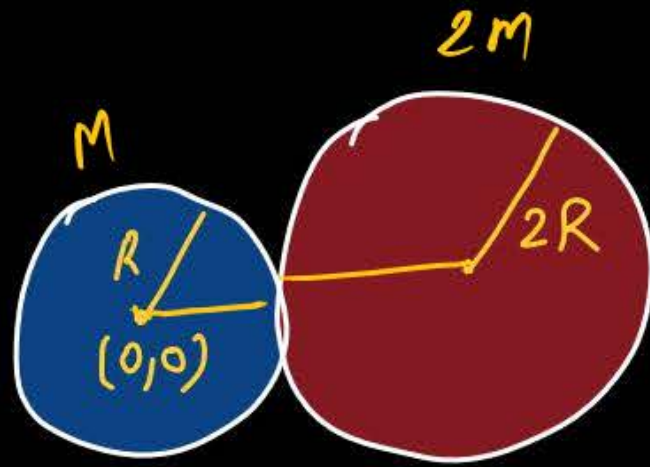
$$= \frac{m \frac{2R}{\pi} - 2m \left(\frac{2R}{\pi} \right)}{m + 2m}$$

Two disc of same density (ρ) then find C.O.M of system.

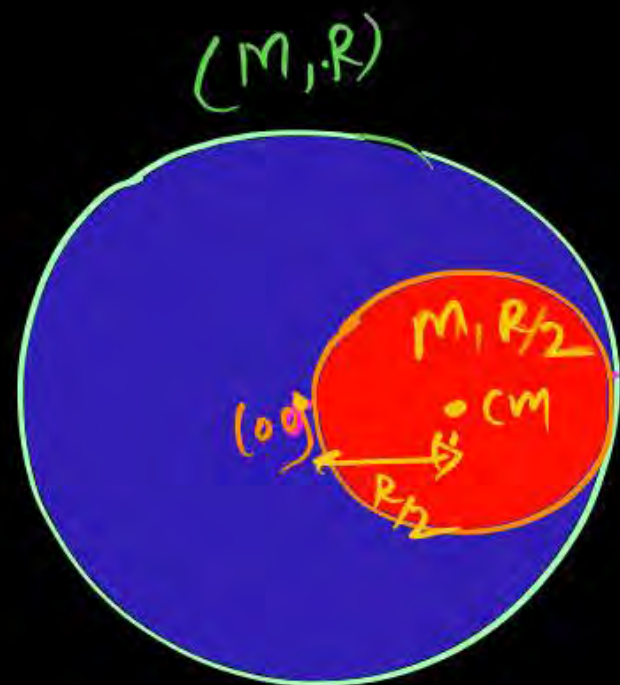


$$x_{cm} = \frac{m_1 \times 0 + m_2 \times 3R}{m_1 + m_2} = \frac{4m \times 3R}{5m} = \frac{12R}{5} \checkmark$$

Two disc of ~~same radii (R)~~
mass m , & $2m$ then find C.O.M of system.



$$x_m = \frac{m \times 0 + 2m(3R)}{3m}$$

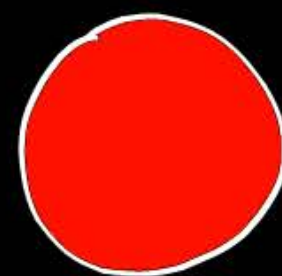
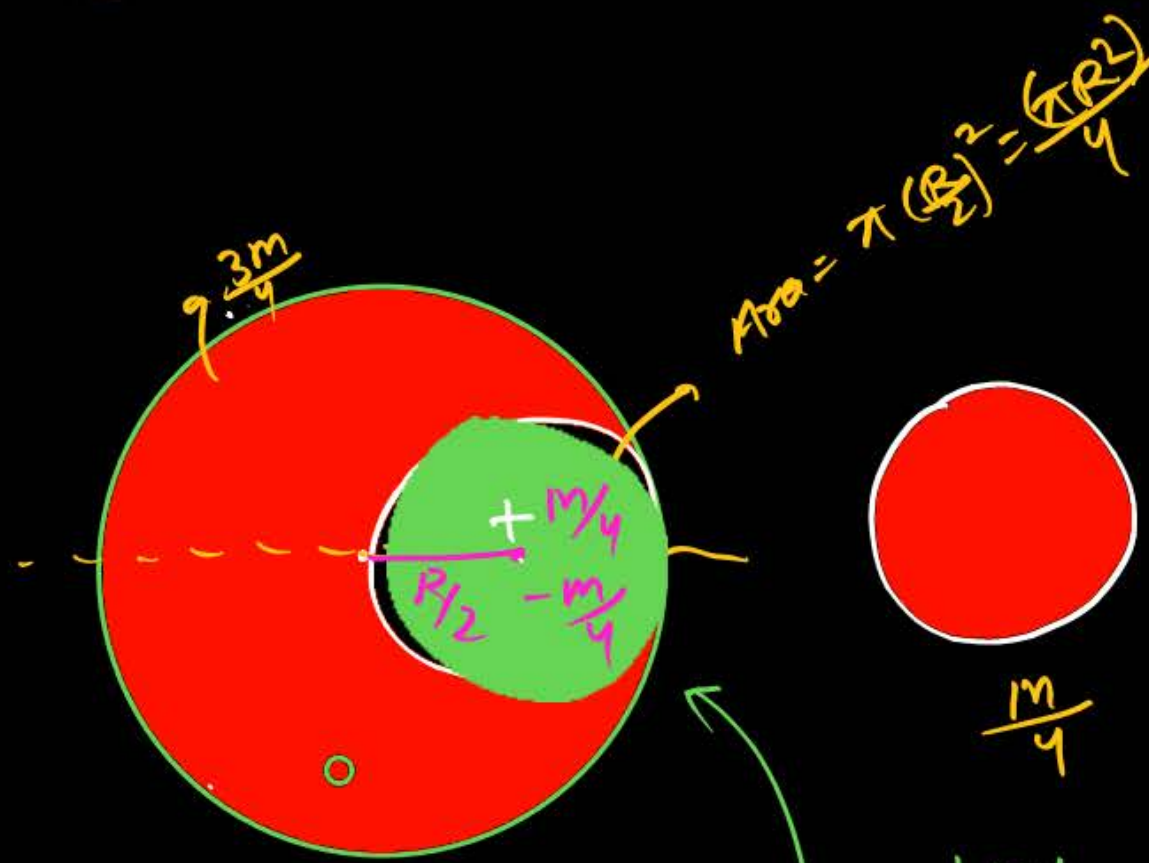
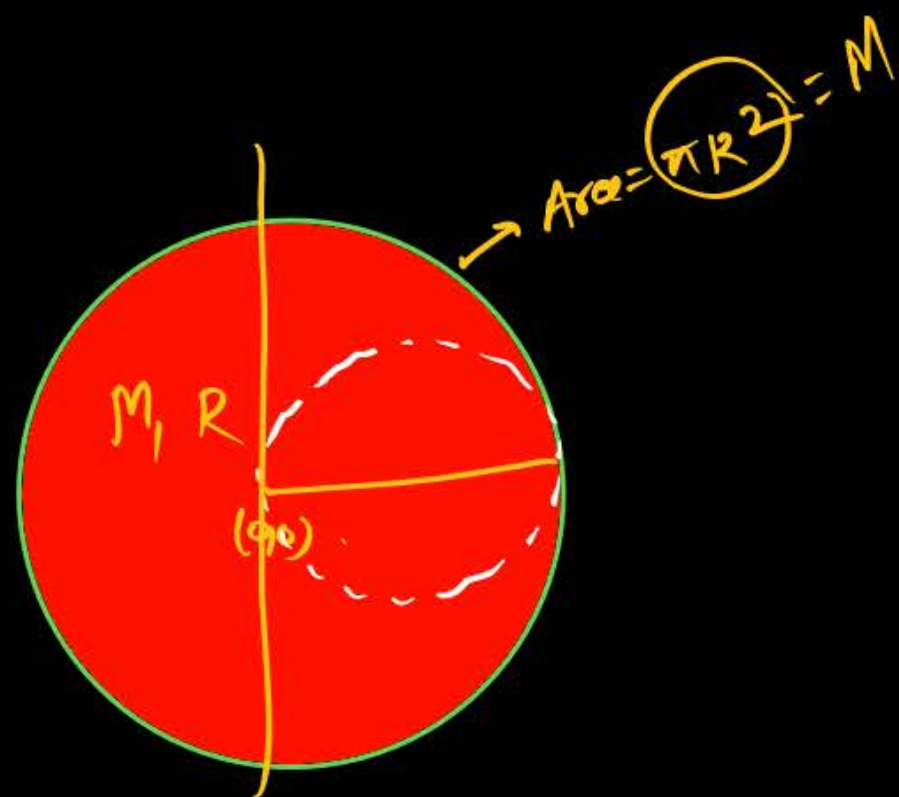


$$\begin{array}{cc} M & m \\ \bullet & \bullet \\ (0,0) & R/2 \end{array}$$

$$x_{cm} = \frac{m \times 0 + m R/2}{2m} = \frac{R}{4}$$

Find C.O.M of remaining part.

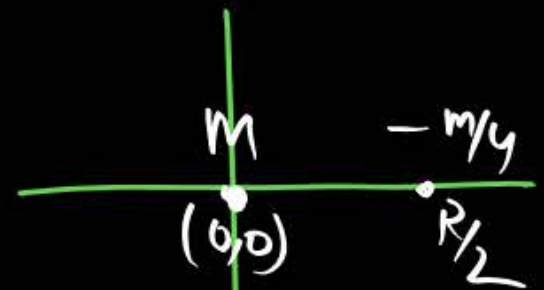
✓
from this



$$\frac{m}{4}$$

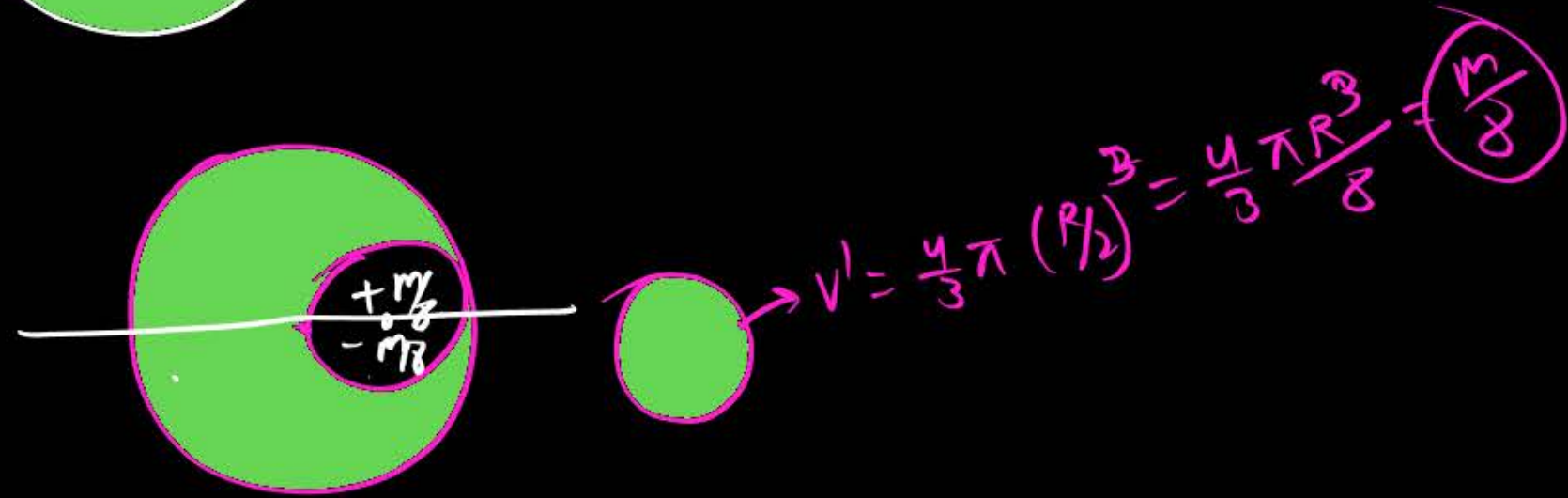
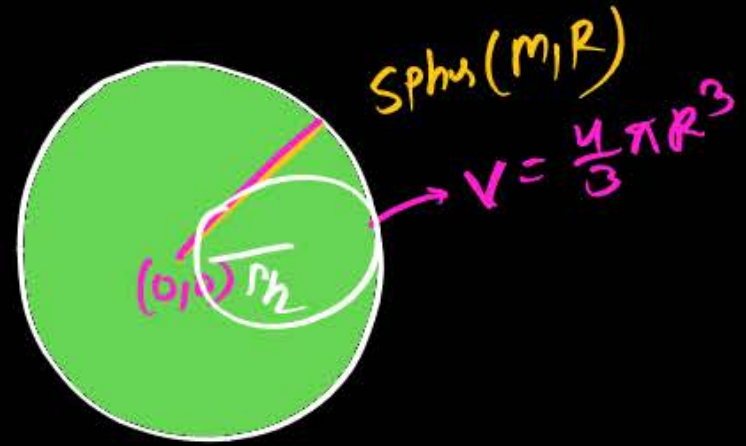
$$+m/4$$

$$-m/4$$



$$x_{cm} = \frac{M \times 0 - \frac{m}{4} R/2}{M - \frac{m}{4}} = \frac{-\frac{m}{4} \times \frac{R}{2}}{\frac{3m}{4}} = -\frac{R}{8}$$

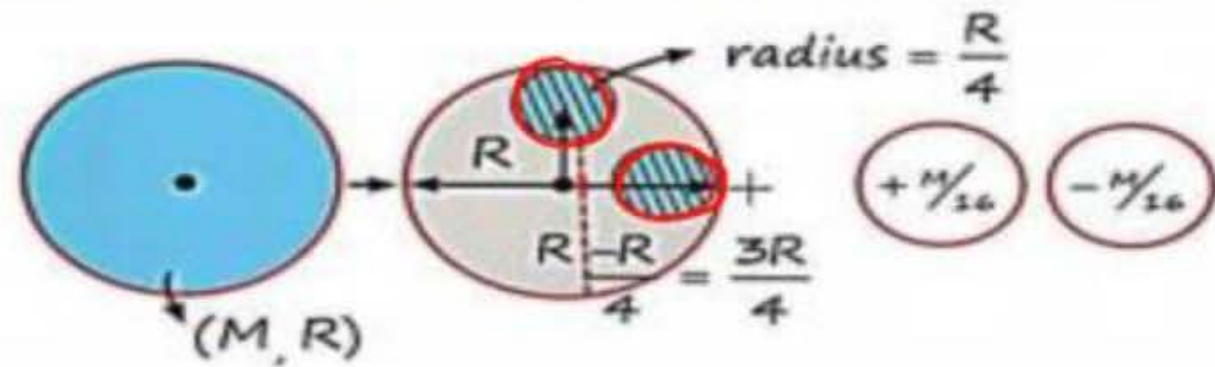
A sphere of mass M and Radius R , A Uniform sphere of Radius $R/2$ is cut then find C.O.M of remaining part.



$\underline{x_{cm} =}$

Diagram showing the center of mass calculation. A horizontal line represents the axis of symmetry. The center of the original sphere is marked with a dot and labeled M . The center of the cut-out sphere is marked with a dot and labeled $-\frac{M}{8}$. The distance between the two centers is labeled $R/2$.

Q.18. From a disc of mass M and radius R , two disc of radius $R/4$ is removed as shown in figure then find position of C.O.M of remaining part:-



Sol. Mass = $\left(\frac{M}{16}\right)$

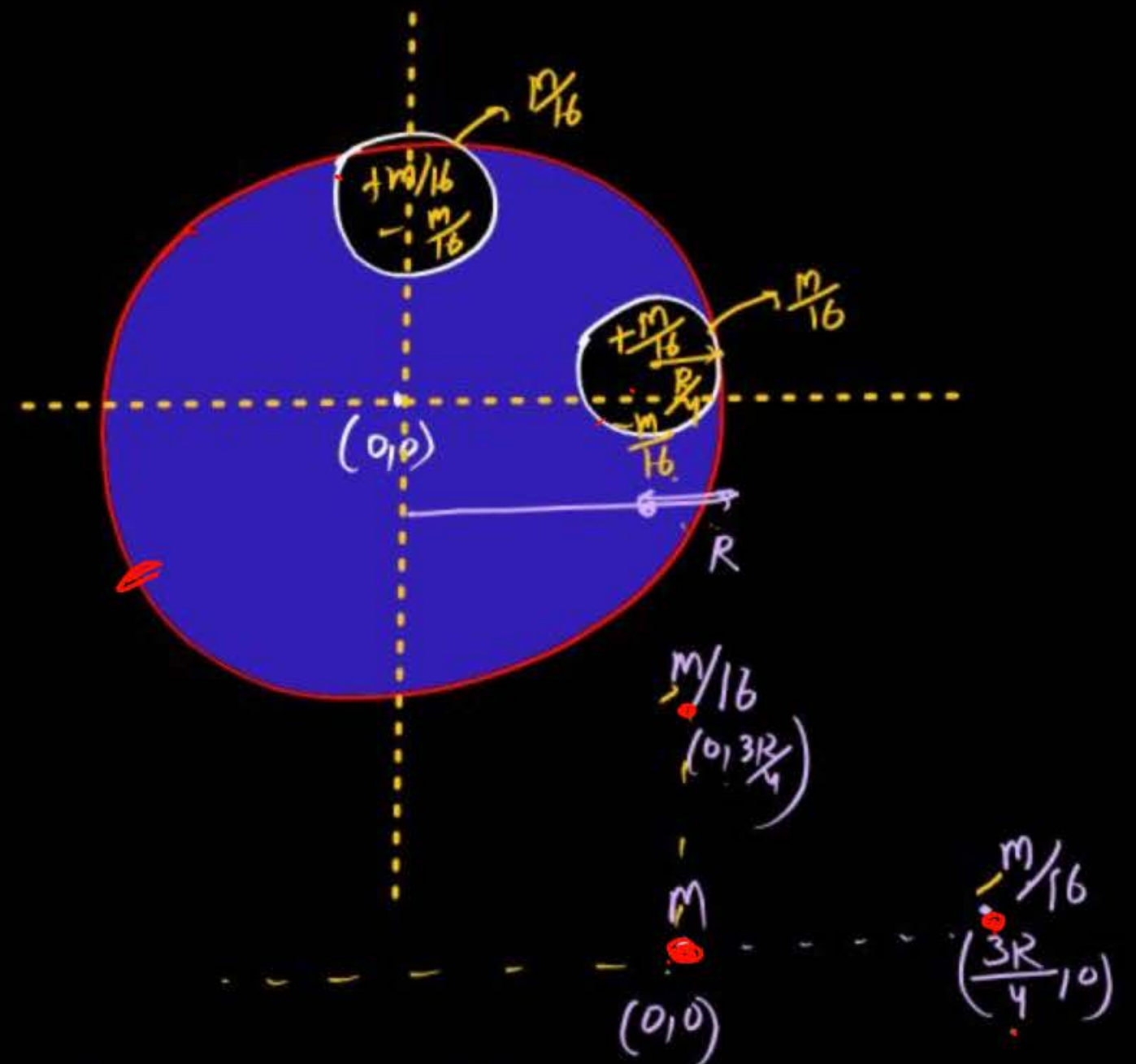
$$\text{Area} = \pi \left(\frac{R}{4}\right)^2 = \frac{\pi R^2}{16}$$

For x-coordinate:

$$x_{c.m} = \frac{M \times 0 - \frac{M}{16} \frac{3R}{4} - \frac{M}{16} \times 0}{M - \frac{M}{16} - \frac{M}{16}}$$

$$x_{c.m} = \frac{-3R}{56}$$

$$r_{c.m} = \left(\frac{-3R}{56} \hat{i}, \frac{-3R}{56} \hat{j}\right)$$



$$x_{(m)} = \frac{m \times 0 - \frac{m}{16} \left(\frac{3R}{4}\right) + \frac{m}{16} \times 0}{M - \frac{m}{16} - \frac{m}{16}}$$

$$= \frac{\frac{3mR}{64}}{\frac{14m}{16}} = \frac{3R}{56}$$

find velocity of c.o.m, momentum of system and K.E of system.

①



$$\vec{v}_{c.m} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$$= \frac{mu\hat{i} + mu\hat{i}}{2m} = \frac{2mu\hat{i}}{2m} = \underline{u\hat{i}}$$

$$P_{\text{system}} = M_{\text{system}} v_{c.m} = 2mu\hat{i}$$

$$K.E_{\text{system}} = M_{\text{system}} \times (v_{c.m})^2 \quad \times$$

$$\vec{p}_{\text{system}} = M_{\text{system}} \cdot \vec{v}_{c.m} \quad \checkmark$$

$$K.E_{\text{system}} = \frac{1}{2} M_{\text{system}} (v_{c.m})^2 \quad \times$$

$$K.E = \frac{1}{2} mu^2 + \frac{1}{2} mu^2 = mu^2 \quad \checkmark$$

②



$$\vec{v}_{c.m} = \frac{mu\hat{i} - mu\hat{i}}{2m} = 0$$

$$\vec{p}_{c.m} = 2m \times 0 = 0$$

$$K.E = \frac{1}{2} (2m) v_{c.m}^2 = 0 \quad \times$$

$$K.E = \frac{1}{2} mu^2 + \frac{1}{2} mu^2 = mu^2 \quad \checkmark$$

$$\vec{v}_{c.m.} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$$(m_1 + m_2) \vec{v}_{c.m.} = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

$$M_{\text{sys}} \vec{v}_{c.m.} = \vec{p}_1 + \vec{p}_2$$

$$\boxed{M_{\text{sys}} \vec{v}_{c.m.} = \vec{p}_{\text{sys}}}$$

$\vec{p}_{\text{sys}}$

③



$$\vec{V}_{cm} = \frac{mu\hat{i} + mu\hat{j}}{2m}$$

$$K.E = \frac{1}{2}mu^2 + \frac{1}{2}m(u)^2 \leftarrow \text{SCAL}$$

④



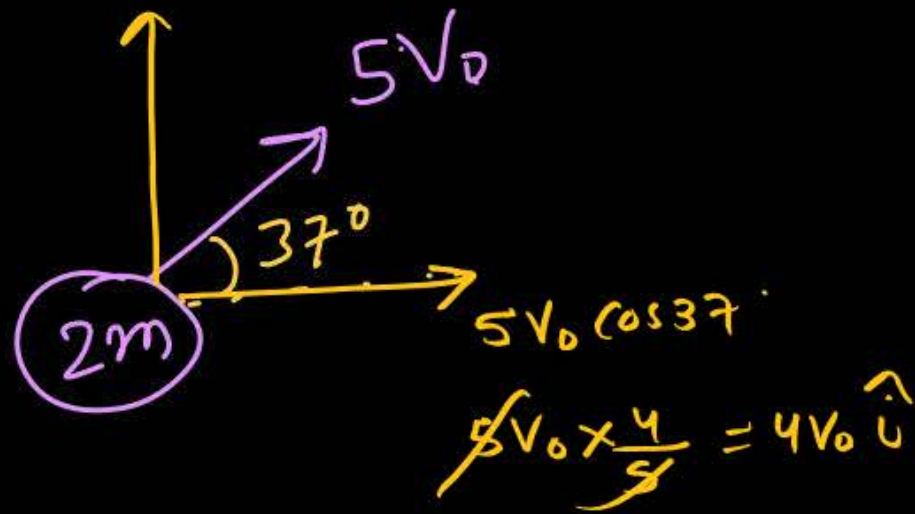
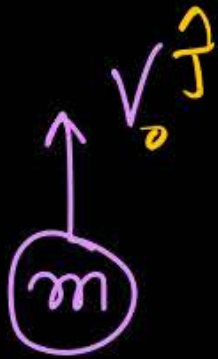
$$V_{cm} = \frac{mv\hat{i} + 2mv\hat{i} - 3mv\hat{i}}{m + 2m + 3m} \checkmark$$

$$\vec{P}_{sys} = 6m \vec{V}_{cm}$$

$$K.E = K.E_1 + K.E_2 + K.E_3 \checkmark$$

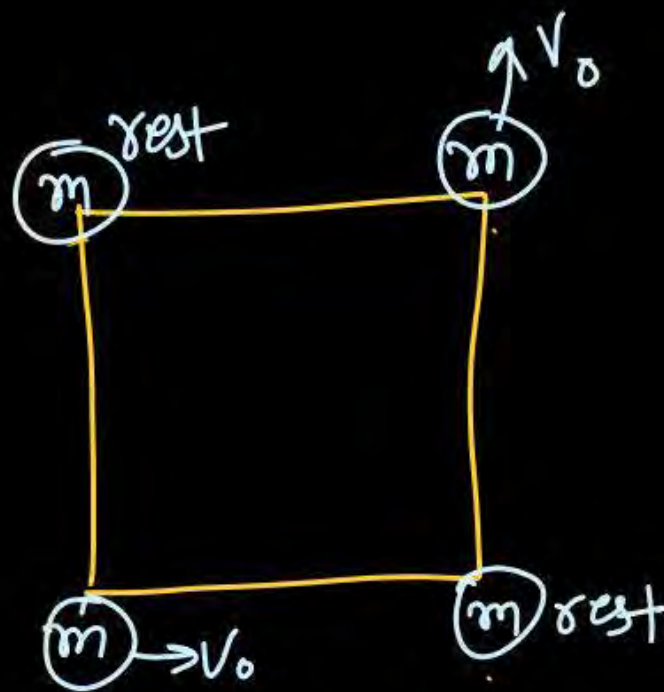
Find velocity of C.O.M.

⑤



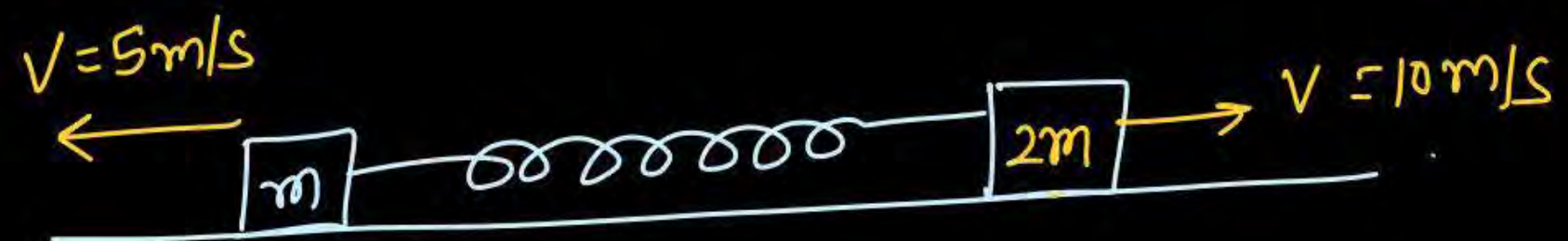
$$V_{cm} = \frac{mv_0 \hat{j} + 2m(4v_0 \hat{i} + 3v_0 \hat{j})}{3m}$$

$$K.E = \frac{1}{2}mv_0^2 + \frac{1}{2}2m(5v_0)^2$$
$$=$$



$$\vec{p}_{c.m.} = \frac{0xv + 0yv + mv_0 + mv_0}{4m}$$

$$\vec{v}_{c.m.} = \frac{v_0\hat{i} + v_0\hat{j}}{4}$$

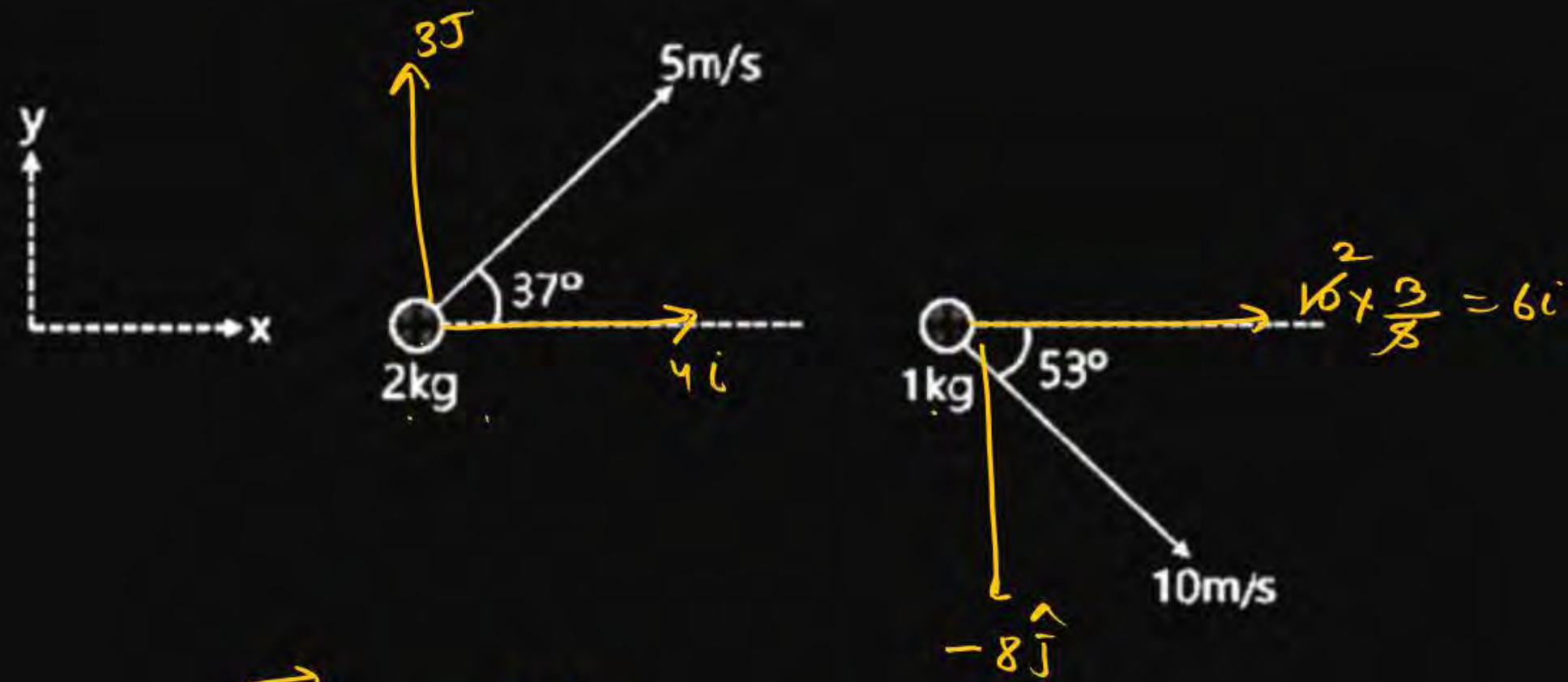


$$v_{cm} = \frac{2m \times 10i - m(5)i}{2m + m}$$

Question



Find velocity of COM.



$$\vec{v}_{\text{COM}} = \frac{2(4\hat{i} + 3\hat{j}) + 1(6\hat{i} - 8\hat{j})}{2+1}$$

Acceleration of centre of mass :-

① $m \rightarrow a_i = a$ $2m$ rest

$$a_{cm} = \frac{ma_i + m \times 0}{2m} = \frac{\cancel{m}a}{\cancel{2m}} = \frac{a}{2} i$$

② $m \rightarrow a$ $\leftarrow \frac{2m}{a}$

$$\begin{aligned} \vec{a}_{cm} &= \frac{\cancel{m}a i - 2\cancel{m}a i}{\cancel{3m}} \\ &= \frac{-a i}{3} \end{aligned}$$

③



$$\begin{aligned} \vec{a}_{cm} &= \frac{-m \times g \hat{j} + 2m(-g \hat{j})}{3m} \\ &= \frac{-\cancel{3m}g \hat{j}}{\cancel{3m}} = -g \hat{j} \end{aligned}$$

Question

NEET/SEE



Two particles of mass 1 kg and 3 kg are having acceleration $(\hat{i} - 2\hat{j} + 5\hat{k})$ m/s² and $(-2\hat{i} + \frac{4}{3}\hat{j} - \hat{k})$ m/s². Find out $\vec{a}_{cm}$.

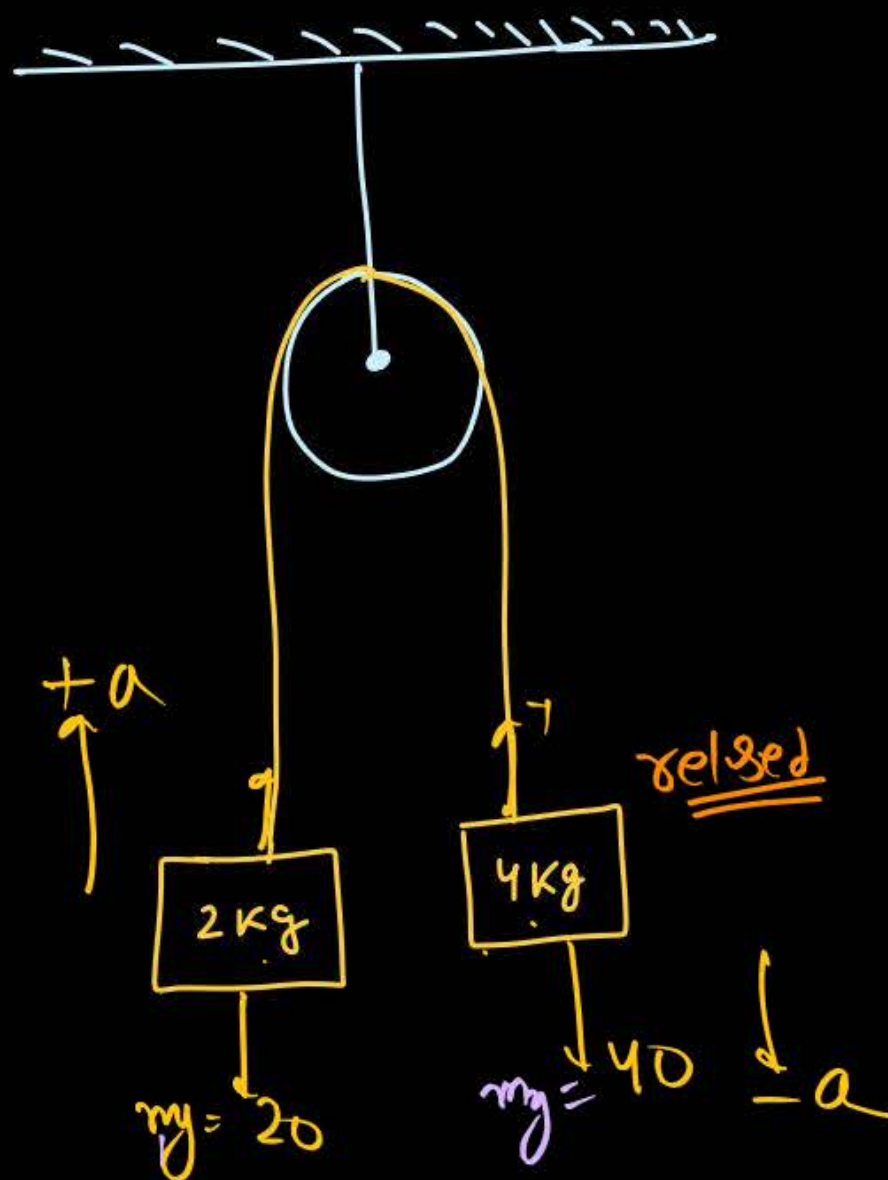
$$a_{cm} = \frac{1(\hat{i} - 2\hat{j} + 5\hat{k}) + 3(-2\hat{i} + \frac{4}{3}\hat{j} - \hat{k})}{1+3}$$

Question



Three particles of masses 1 kg, 2 kg and 3 kg are subjected to forces $(3\hat{i} - 2\hat{j} + 2\hat{k})\text{N}$, $(-\hat{i} + 2\hat{j} - \hat{k})\text{N}$ and $(\hat{i} + \hat{j} + \hat{k})\text{N}$ respectively. Find the magnitude of the acceleration of the CM of the system.

Find accⁿ of C.O.M.



$$a = \frac{40 - 20}{6} = \frac{20}{6} = \frac{10}{3} \text{ m/s}^2$$

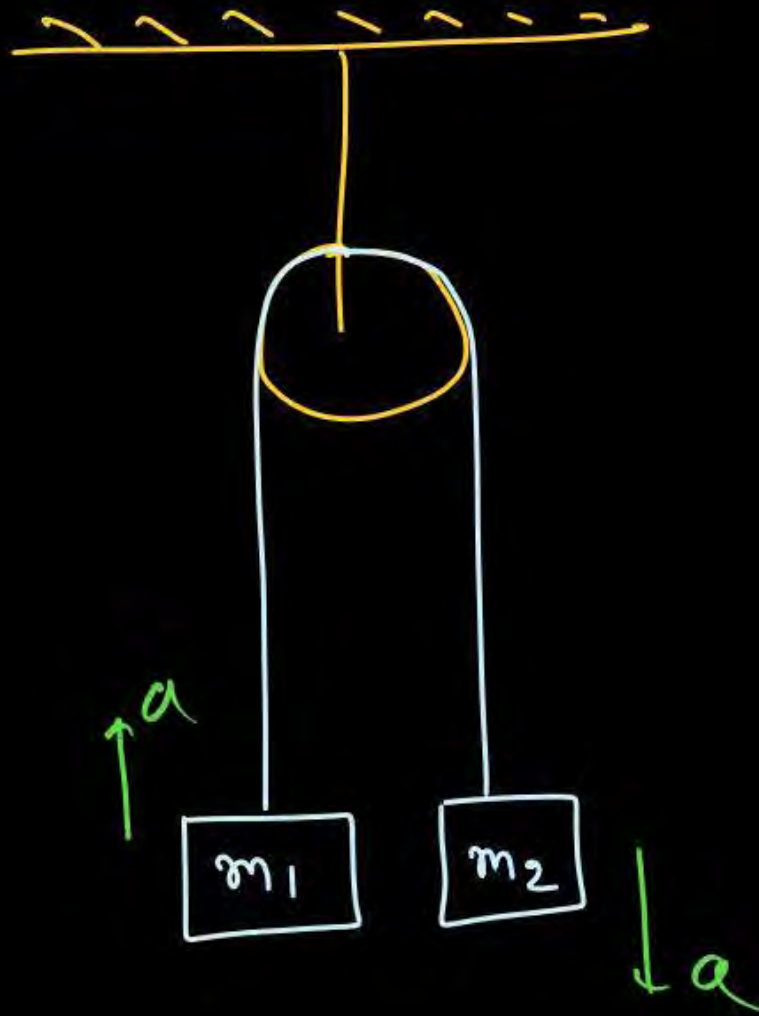
$$\vec{a}_{\text{C.M.}} = \frac{4\left(-\frac{10}{3}\hat{j}\right) + 2\left(\frac{10}{3}\hat{j}\right)}{6}$$

$$\checkmark = \frac{(-4+2) \frac{10}{3} \hat{j}}{6}$$

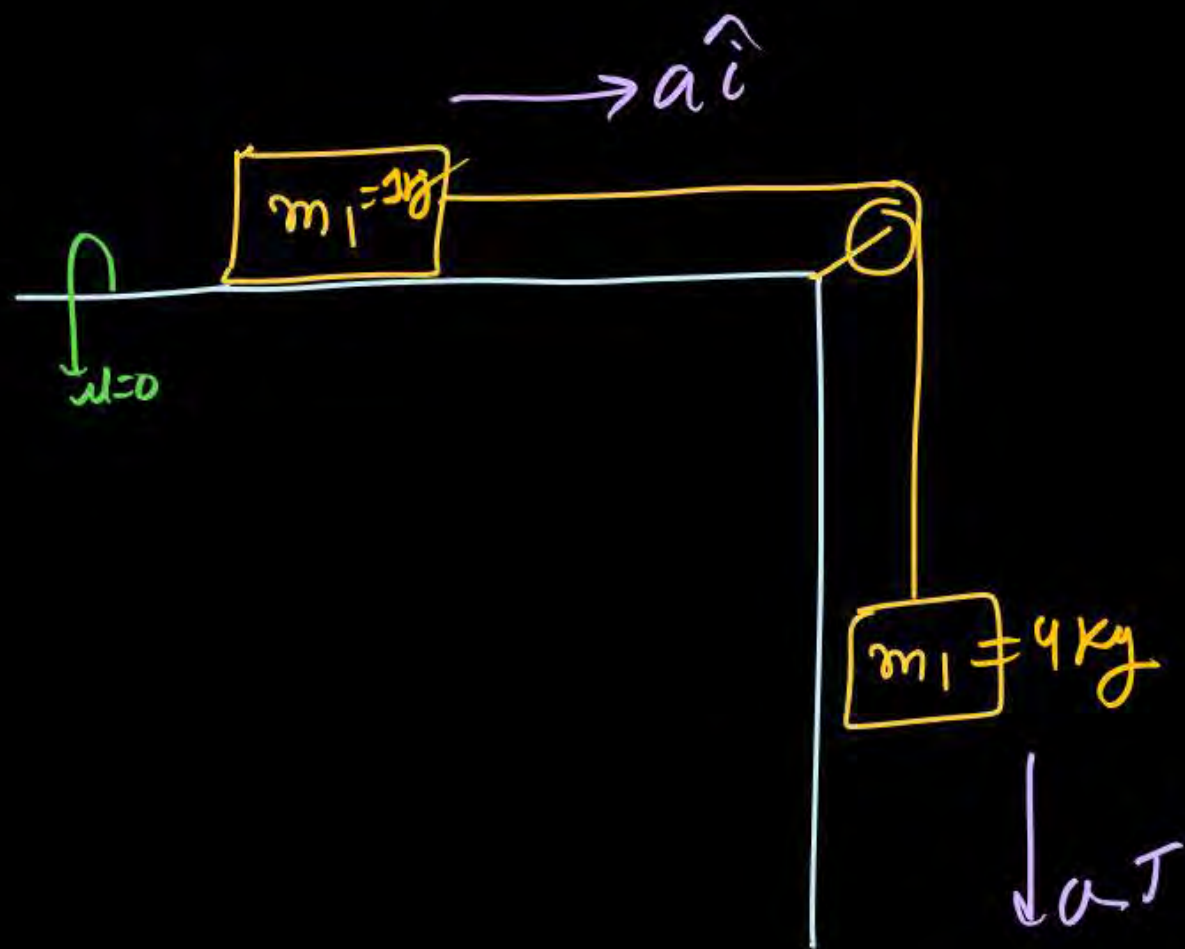
$$= -\frac{2}{3} \times \frac{10}{3} = -\frac{10}{9} \text{ m/s}^2$$

$$\begin{aligned} \text{II} \quad a_{\text{C.M.}} &= \left(\frac{m_2 - m_1}{m_2 + m_1} \right)^2 g \\ &= \left(\frac{4-2}{4+2} \right)^2 \times 10 = \left(\frac{2}{6} \right)^2 \times 10 = \frac{10}{9} \end{aligned}$$

find accn of C.O.M



$$a_{cm} = \left(\frac{m_2 - m_1}{m_2 + m_1} \right)^2 g$$

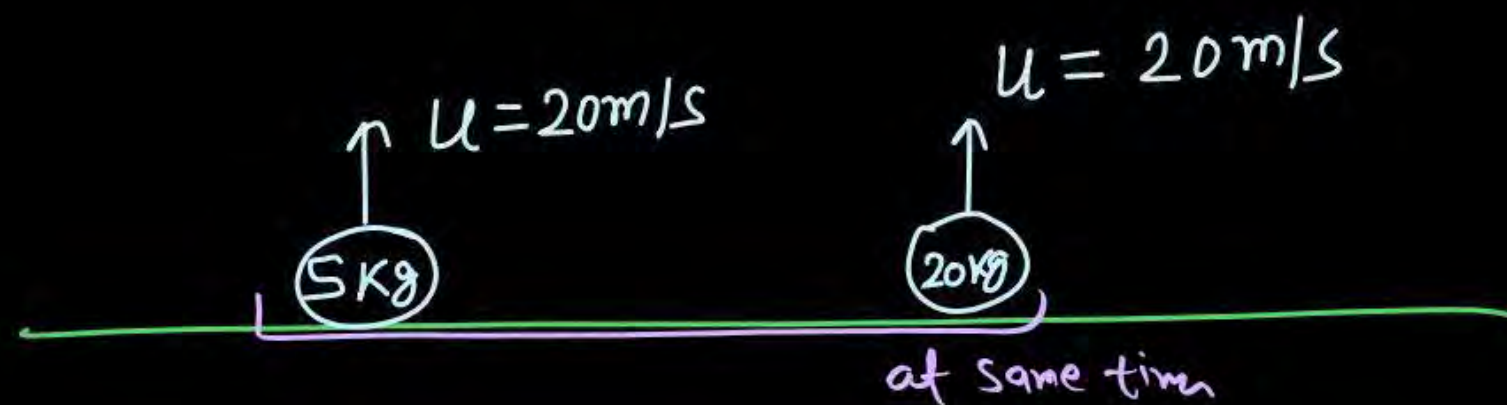


find accⁿ of C.O.M.

$$\vec{a} = \frac{40 - 0}{5} = \frac{40}{5} = 8 \text{ m/s}^2$$

$$\vec{a}_{\text{cm}} = \frac{1 \times 8 \hat{i} + 4(-8 \hat{j})}{5}$$

find maximum height attained by the C.O.M



$$H_{\text{C.M}} = ??$$

$$H_{\text{C.M}} = \frac{u_{\text{C.M}}^2}{2a_{\text{C.M}}} = \frac{(20)^2}{2 \times 10} = \frac{400}{20} = \underline{\underline{20\text{cm}}}$$

$$\vec{u}_{\text{C.M}} = \frac{5 \times 20\hat{j} + 20 \times 20\hat{j}}{25}$$

$$= \frac{100\hat{j} + 400\hat{j}}{25} = \frac{500}{25} = 20\hat{j}$$

$$a_{\text{C.M}} = \frac{5 \times g\hat{j} + 20 \times (-g\hat{j})}{25}$$

$$\vec{a}_{\text{C.M}} = \frac{-25g\hat{j}}{25} = -g\hat{j}$$

Conservation of Linear momentum of system

for point object

$$F = \left(\frac{dP}{dt} \right)$$

$$\begin{aligned} m \rightarrow \vec{v} \\ \vec{p} = m\vec{v} \end{aligned}$$

$$P = Ct^n \rightarrow F = 0$$

$$\text{if } F = 0 = \frac{dP_{\text{system}}}{dt}$$

$$P = Ct^n$$

obv

for system of particle



$$\vec{F}_{\text{ext}} = \frac{d(P_{\text{system}})}{dt}$$

$$\vec{F}_{\text{ext}} = \frac{d(m\vec{v}_{\text{cm}})}{dt}$$

$$\vec{F}_{\text{ext}} = m \frac{d(\vec{v}_{\text{cm}})}{dt}$$

$$\vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$$

$$\vec{a}_{\text{cm}} = \frac{F_{\text{ext}}}{M_{\text{system}}}$$

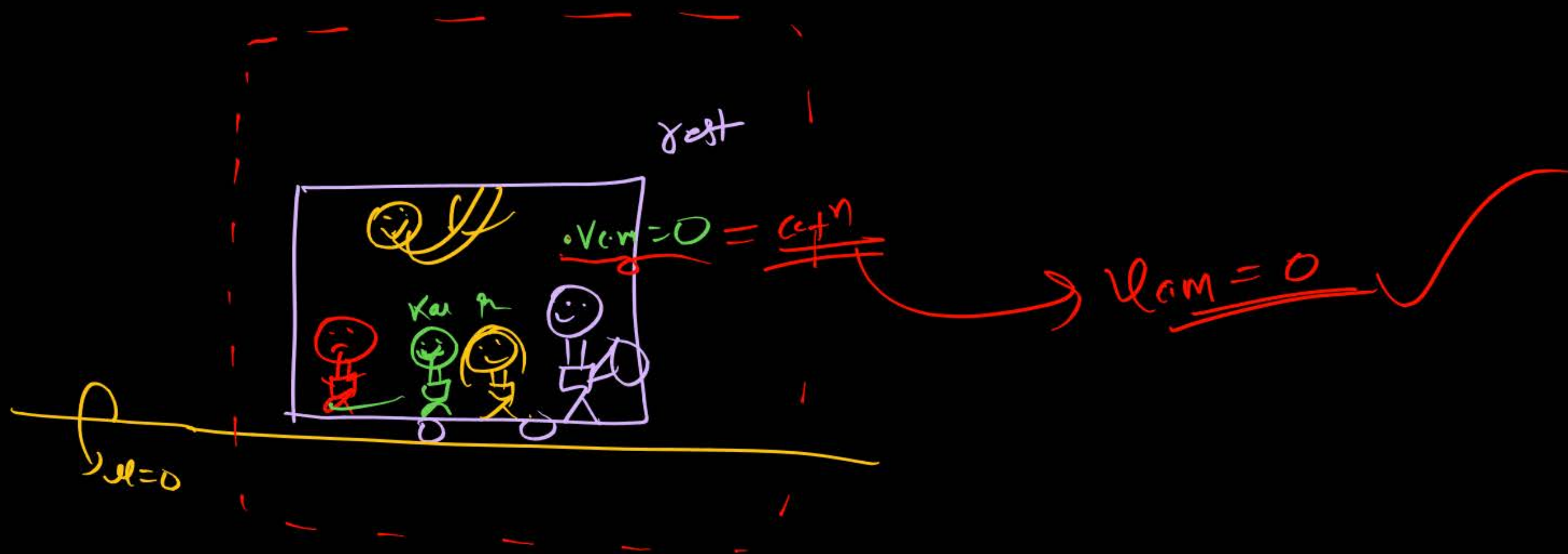
$$\Rightarrow F_{\text{ext}} = 0 = \frac{d(P_{\text{system}})}{dt}$$

$$\vec{P}_{\text{sys}} = Ct^n$$

$$M\vec{v}_{\text{cm}} = Ct^n$$

$$\vec{v}_{\text{cm}} = Ct^n$$

$$\vec{a}_{\text{cm}} = 0$$



MCQ

If external force is zero then location of centre of mass will not change $\rightarrow$ False ✓

If $f_{\text{ext}} = 0$ then state of motion of COM will not change. [$\vec{V}_{\text{cm}} = \text{const}^n$]
 $a_{\text{cm}} = 0$ ✓

- ✓ aag lge chahе Bati me C.O.M. rahe
apne masti me
(Stuck) $V.C.M = 0$

MR* DOX

Question me object
hai ya system ki
bat kar Raha hai
dhyam se Padho.

System or C.O.M ka Momentum
external force se hi change hota hai
internal force ki aulad Nahi hai
System ki moment ko change karke ✓

→ but interter force / external
dono hi system ki K.E. ko
change kar skta hai.

statement

- (1) momentum of system only change when external force acting on system = true ✓
- (2) K.E. of system only change when external force is acting false
- (3) if $f_{ext} = 0$ on system then moment of object conserved $\rightarrow$ false
- (4) if centre of mass is rest then all particle must be at rest $\rightarrow$ false



$$U_{cm} = 0 \quad P_{cm} = 0$$

⑤ gf $F_{ext} = 0$ then position of C.O.M will not change $\rightarrow$ False

⑥ gf K.E of system is zero momentum of system must be zero. $\rightarrow$ True
all are at rest

⑦ gf momentum of system is zero then K.E must be zero $\rightarrow$ false
 $(m) \xrightarrow{u} \quad \xleftarrow{u} (m) \quad P_{C.M} = 0$

⑧ gf all particle at rest then C.O.M must be at rest $\rightarrow$ True

⑨ system can ^{may} have K.E without linear momentum $\rightarrow$ True



⑩ system may have momentum without having K.E: false

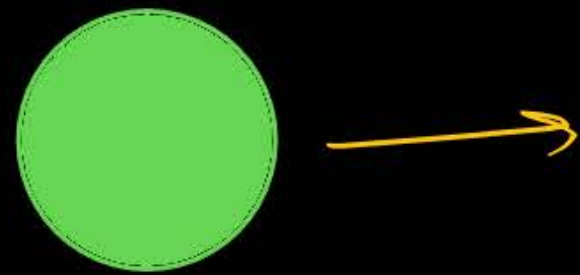
$$K.E_{\text{system}} = 0 = \underbrace{\frac{1}{2} m_1 v_1^2}_{\text{all are at rest}} + \underbrace{\frac{1}{2} m_2 v_2^2} + \underbrace{\frac{1}{2} m_3 v_3^2}$$

⑪ internal force can't change momentum & K.E of system $\rightarrow$ false

⑫ if external force is zero then them may be variable $\rightarrow$ false

Explosion $\rightarrow (F_{ext})_{system} = 0$ it happen due to impulsive internal force

- ① Object of mass m is at rest Break into Two Part then which of the following diagram of velocity is correct



M, rest
 $v=0$
 $\underline{P_i=0}$

$\underline{F_{ext}=0}$

$\underline{\vec{P}_f = \vec{P}_i = 0}$

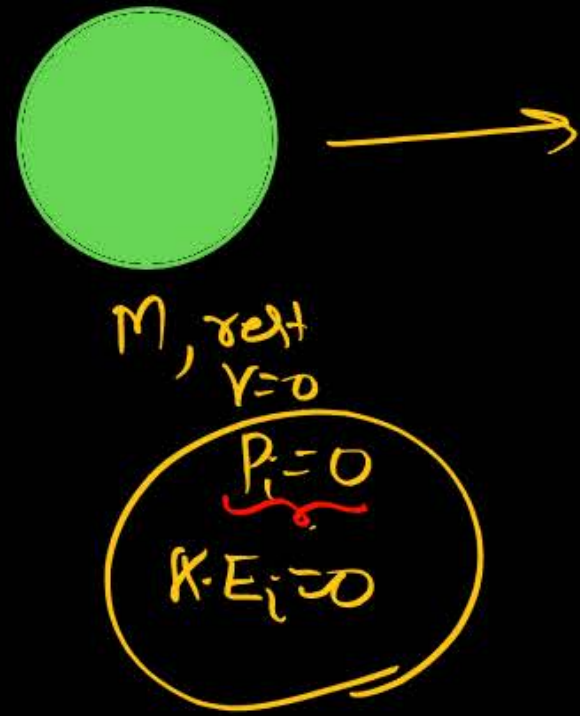


~~Not Possible~~



Explosion $\rightarrow (F_{ext})_{system} = 0$ it happen due to impulsive internal force

- ① Object of mass m is at rest Break into Two Part then which of the following diagram of velocity is correct



$$\vec{P}_f = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

$$F_{ext} = 0$$

$$\vec{P}_i = \vec{P}_f$$

$$0 = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

$$m_1 \vec{v}_1 = -m_2 \vec{v}_2$$

$$\vec{P}_1 = -\vec{P}_2 \quad \checkmark$$

$$\textcircled{1} \frac{P_1}{P_2} = 1:1$$

$$\textcircled{2} \frac{v_1}{v_2} = \frac{m_2}{m_1}$$

$$\textcircled{3} \vec{v}_1 = -\frac{m_2 \vec{v}_2}{m_1}$$

$$\textcircled{4} \frac{K.E_1}{K.E_2} = \frac{m_2}{m_1} \quad \neq$$

$$K.E = \frac{p^2}{2m}$$

⑤ Work done by internal

$$W_{int} = \Delta K.E$$

$$= \frac{1}{2} m_1 (v_1)^2 + \frac{1}{2} m_2 (v_2)^2$$

Question



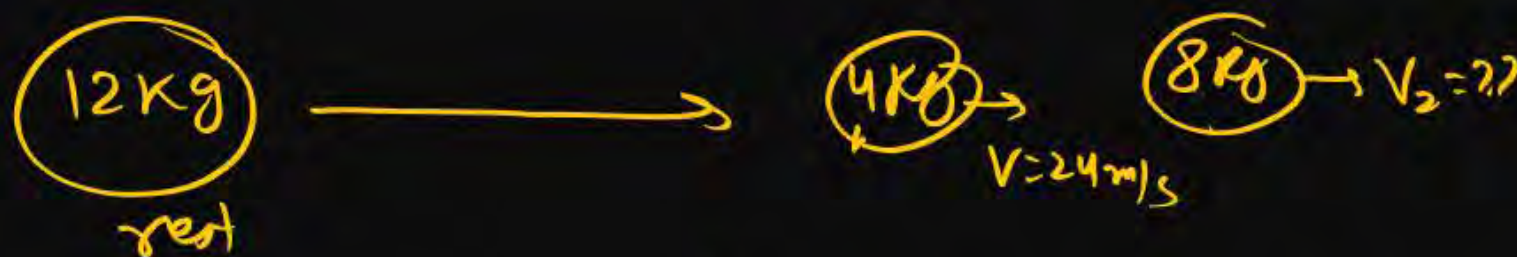
A bomb of mass 12 kg explodes into two pieces of 4 kg and 8 kg. The velocity of 4 kg piece is 24 m/s. The kinetic energy of 8 kg piece is:

1 192 J

2 786 J

3 576 J

4 687 J



$$P_i = 0 = 4 \times 24 + 8 v_2$$

$$- \frac{3}{2} \times 4 = 8 v_2$$

$$\boxed{-12 \text{ m/s} = v_2}$$

$$K.E = \frac{1}{2} \times 8 \times (12)^2$$

$$= 4 \times 144$$
$$= 576 \text{ J}$$

$$K.E = \frac{p^2}{2m}$$
$$= \frac{(4 \times 24)^2}{2 \times 8} = 576 \text{ J}$$

Question



At ^{238}U , nucleus decays by emitting an alpha particle of speed v m/s. The recoil speed of the residual nucleus is (in m/s) **[CBSE PMT 1995]**

1 $-4u/234$

2 $v/4$

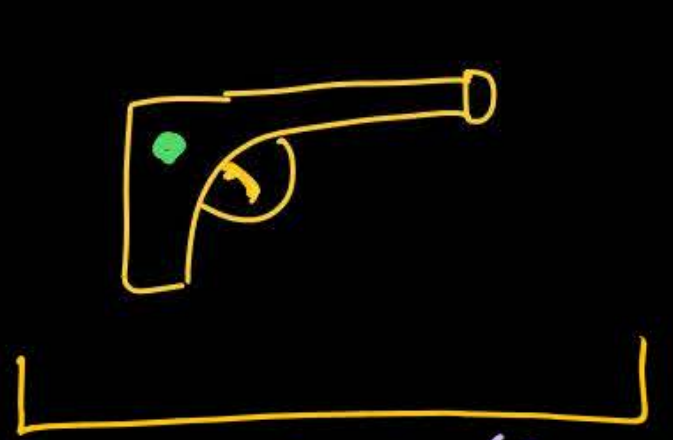
3 $-4v/238$

4 $4v/238$

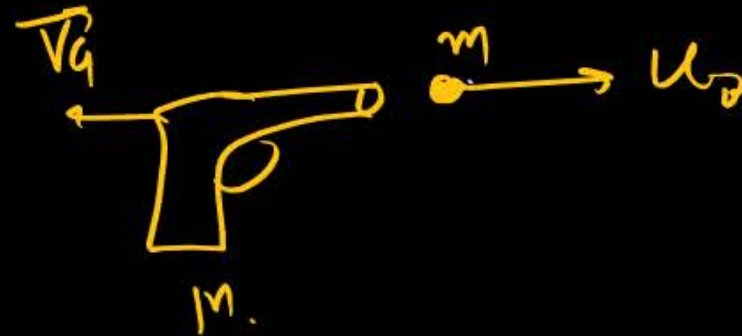
Gun - Bullet system

M = mass of Gun
 V_g = velocity recoil of Gun

m = mass of bullet
 u_b = velocity of bullet



Rest ($P_i = 0$)
 $F_{ext} = 0$
 $\neq$



$$P_f = 0 = \vec{P}_B + \vec{P}_G$$

$$\vec{P}_G = -\vec{P}_B$$

$$M \vec{u}_G = -m \cdot u_b$$

$$\vec{u}_G = -\frac{m \vec{u}_b}{M}$$

⑧ If n -bullets fire per sec, force on gun

$$F = \frac{n(m u_b)}{t} = n m u_b$$

$$\textcircled{1} \vec{P}_{Gun} = -\vec{P}_{bul}$$

$$\textcircled{2} \frac{P_{Gun}}{P_{bul}} = 1:1$$

$$\textcircled{3} \vec{u}_G = -\frac{m \vec{u}_b}{M}$$

$$\textcircled{3} \frac{|u_G|}{|u_B|} = \frac{m}{M}$$

$$\textcircled{4} K.E = \frac{p^2}{2m} \quad K.E \propto \frac{1}{m}$$

$$\textcircled{5} \frac{K.E_B}{K.E_G} = \frac{m_G}{m_B}$$

$$\textcircled{6} K.E_{bul} > K.E_G$$

$$\textcircled{7} W_{int, f} = K.E_B + K.E_G \checkmark$$

Question

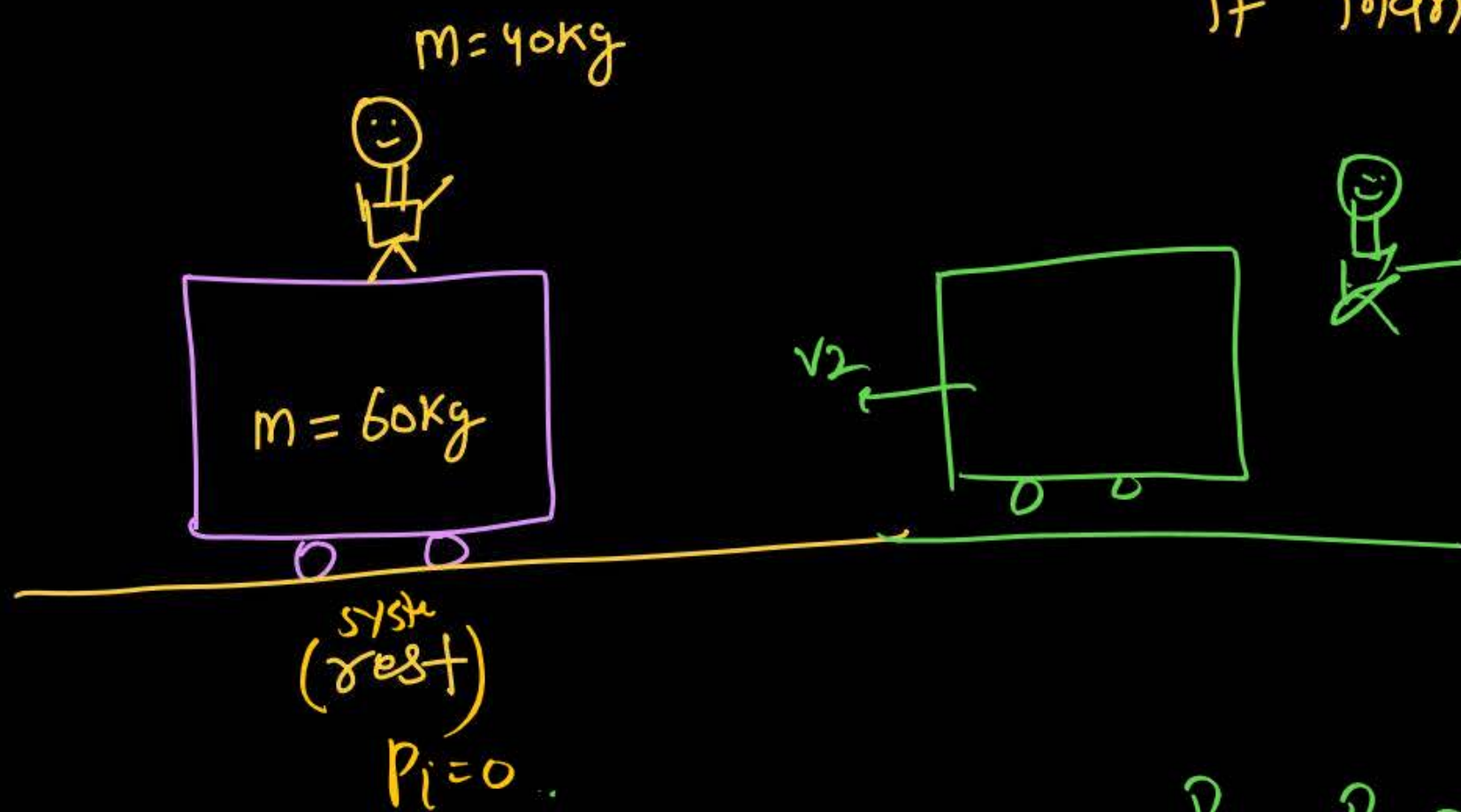


A machine gun fire bullets of mass 40 gm each with the velocity of 100 m/sec. If average force is exerted on the gun F is 400 N, then find the no. of bullets fired per minutes.

$$F = n m u$$

$$400 = n \times \frac{40}{1000} \times 100$$

$$\boxed{n = 10}$$



if man suddenly jump with horizontal velocity 30 m/s then find velocity of cart. also find total energy produced by man.

$$\underline{F_{\text{ext}} = 0}$$

$$P_f = P_i = 0$$

$$m_m v_m + m_B v_B = 0$$

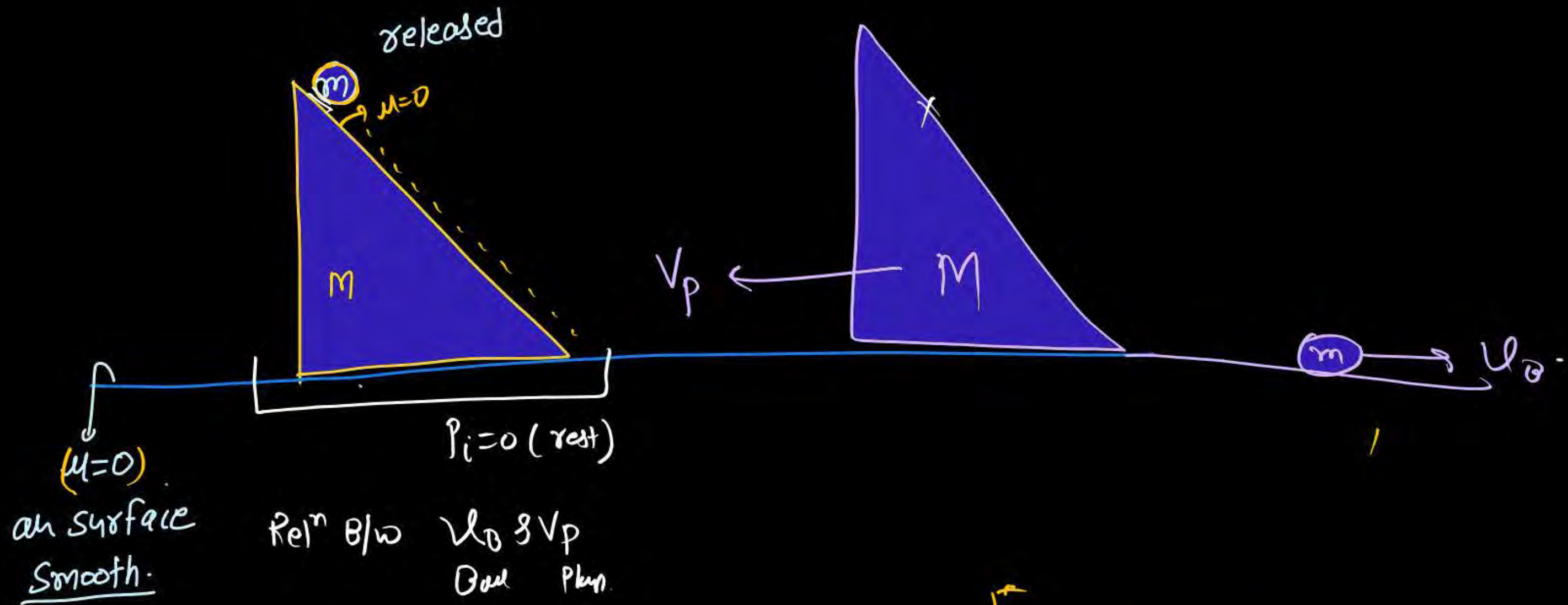
$$40 \times 30 + 60 v_B = 0$$

$$\cancel{60} v_B = - \cancel{40} \times \cancel{30}$$

$$\boxed{v_B = -20} \text{ m/s}$$

$$(K.E)_{\text{sys}} = W_{\text{sys}}$$

$$= \frac{1}{2} 40 (30)^2 + \frac{1}{2} 60 (20)^2$$



$\mu=0$
on surface
Smooth.

Relⁿ B/w u_0 & v_p
Goal Plan.

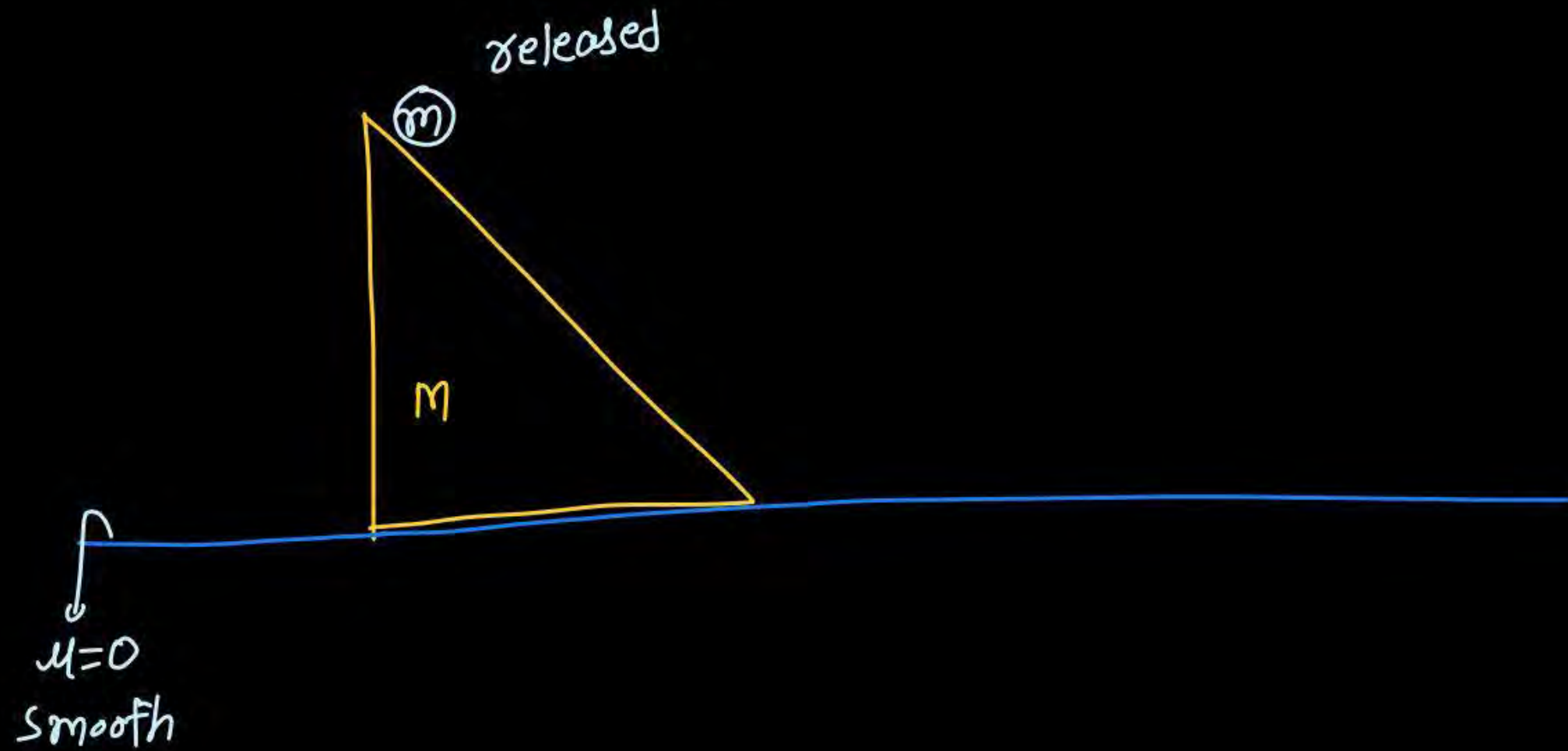
$F_{ext} = \text{gravity (vertical)}$

$F_{Horizontal} = 0$

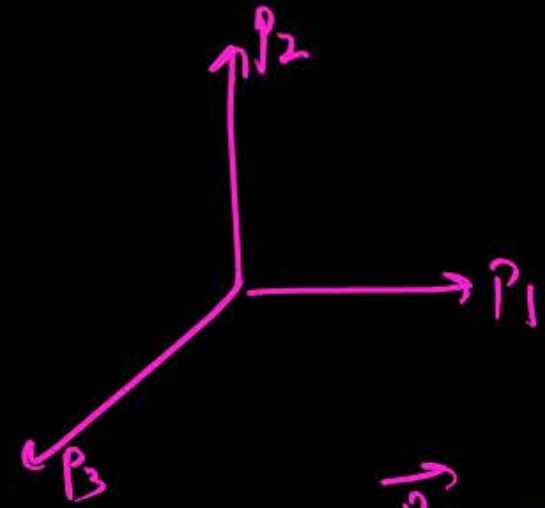
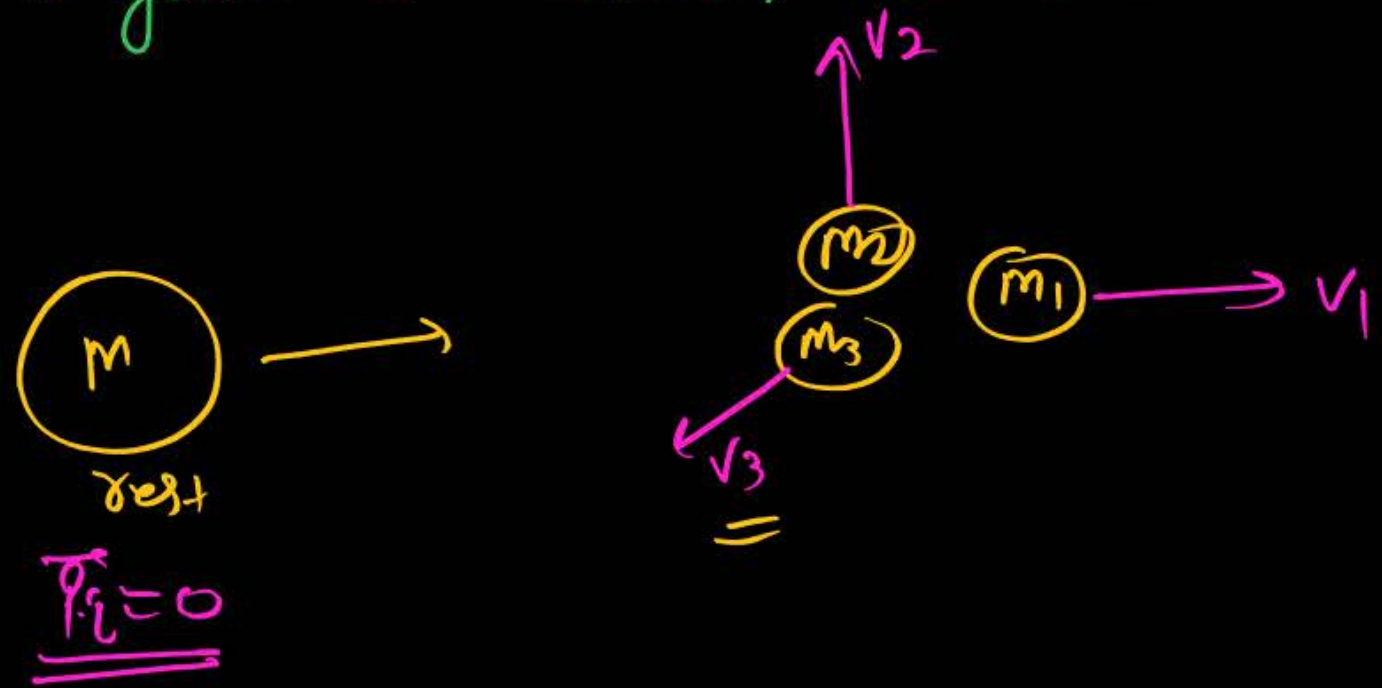
$P_f = P_i = 0$ in x-axn

$$0 = m u_0 + M v_p$$

friction is present only between block and Inclined plane then find
Relation between speed of block and Inclined
plane.

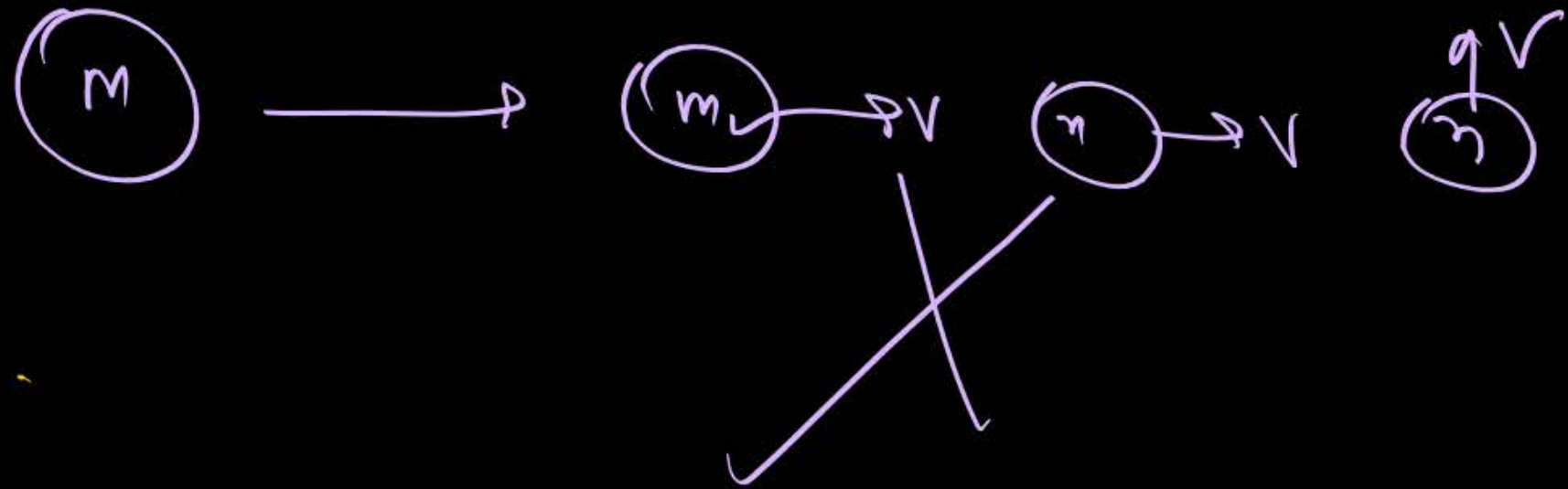


① Object of mass m is at rest → Break into 3 parts then which of the following diagram of velocity is correct



$$\vec{p}_f = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$$

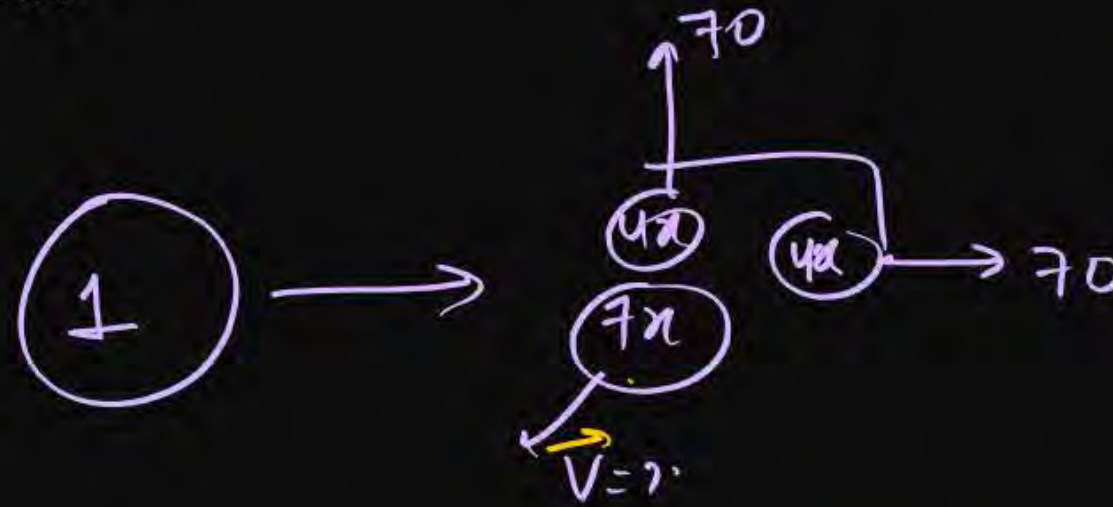
$$\vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$$



Question

A 1 kg stationary bomb is exploded in three parts having mass ratio 4 : 4 : 7. Parts having same mass move in perpendicular directions with velocity 70 m/s, then the velocity of bigger part will be:

- 1 $40\sqrt{2}$ m/s ✓
- 2 $40/\sqrt{2}$ m/s
- 3 $20\sqrt{2}$ m/s
- 4 $20/\sqrt{2}$ m/s



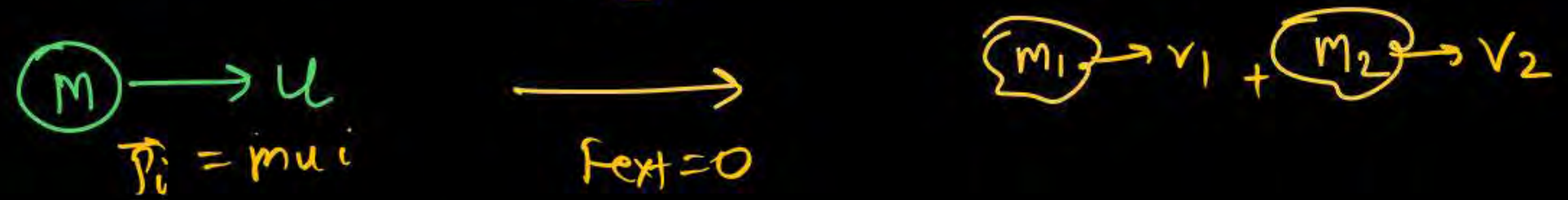
$$m_1 : m_2 : m_3 = 4 : 4 : 7$$
$$= 4x : 4x : 7x$$

$$P_f = 4x \times 70 \hat{i} + 4x \times 70 \hat{j} + 7x \vec{V}$$

$$7x \vec{V} = -[280x \hat{i} + 280x \hat{j}]$$

$$\cancel{7x} \vec{V} = \cancel{-280x} \sqrt{2}$$
$$\vec{V} = \underline{40\sqrt{2}}$$

explosion of moving object :-



$$p_i = p_f$$

$$m\vec{u}\hat{i} = m_1v_1\hat{i} + m_2\vec{v}_2$$

THANK
YOU

MISSION

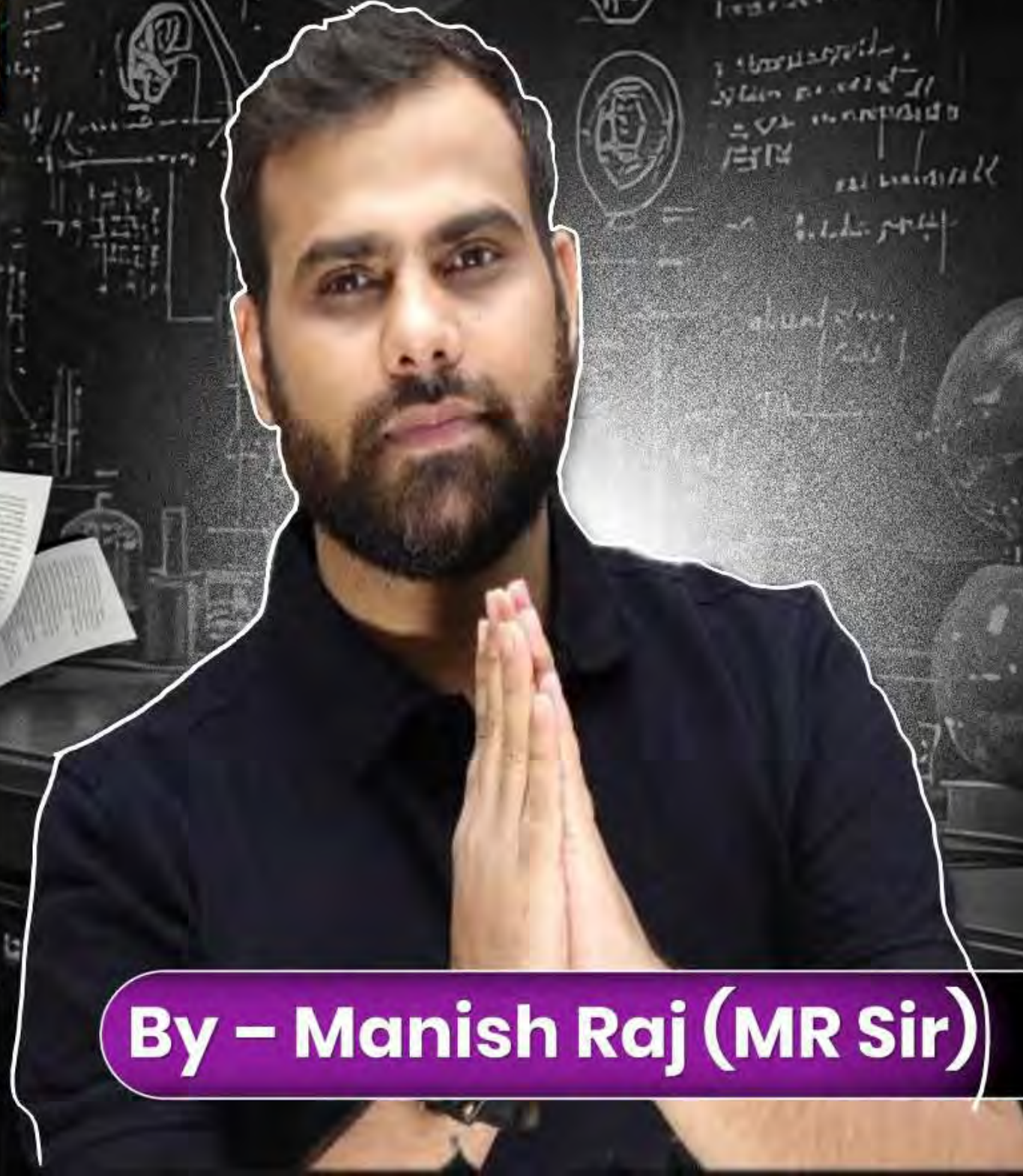
100

Centre of Mass and System of Particles

Physics

Lecture – 03

By – Manish Raj (MR Sir)



Today's Goal

PhD on collision



$$\vec{p}_i = \vec{p}_f \quad \text{system} \checkmark$$

$$M\vec{u} = m_1\vec{v}_1 + m_2\vec{v}_2 \quad \text{---(11)} \checkmark$$

Collision → collision is an interaction b/w two or more than two object, in which external force is zero, but acts large amount of internal force b/w objects.

→ Linear Momentum of system conserved.



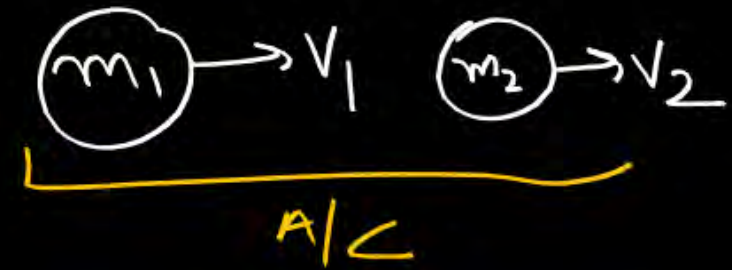
X $u_2 > u_1$ No collision

$\vec{u}_1 > \vec{u}_2$ collision ✓



$F_{\text{ext}} = 0$

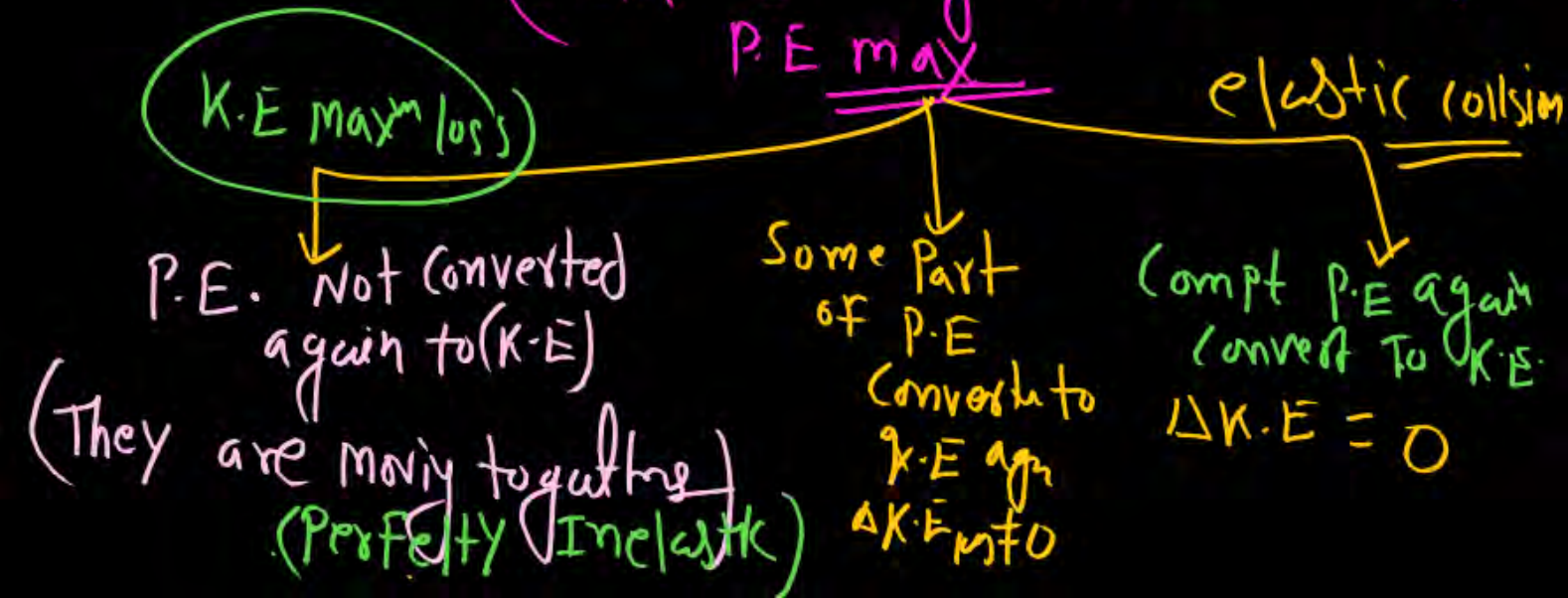
$$m_1 \vec{u}_1 + m_2 \vec{u}_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2 \quad \text{--- (1)}$$



Relative velocity approach (b/c) $= \vec{u}_1 - \vec{u}_2$

Relative velocity of separation $= \vec{v}_2 - \vec{v}_1$

Some part of K.E converted to P.E. (when $v_{\text{rel}} = 0$ moving with same speed)



Collision.

Perfectly Inelastic

(they stick.) & move together

$\Delta K.E_{loss} \rightarrow \text{max}$

Inelastic

Some portion of P.E remain there

$\Delta K.E_{loss} \neq 0$
Not max.

Perfectly elastic

$$\Delta K.E = 0$$

$$K.E_{B/C} = K.E_{A/C}$$

→ Compt P.E.
is recovered
to K.E.

- ① Momentum of system conserved in every type of collision at all the time $\rightarrow$ True
- ② Momentum or K.E. of object may or may not conserved $\rightarrow$ True
- ③ K.E. of system before and after collision conserved in elastic collision $\rightarrow$ True

④ $m_1 \vec{u}_1 + m_2 \vec{u}_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2$

⑤ $e = \frac{(\text{relative velocity of separation}) A/c}{(\text{relative velocity of approach}) B/c} = \frac{\vec{v}_2 - \vec{v}_1}{\vec{u}_1 - \vec{u}_2} = \frac{\vec{v}_1 - \vec{v}_2}{\vec{u}_2 - \vec{u}_1}$

Coefficient of
restitution

Perfectly elastic

$$(\vec{P}_i)_{\text{system}} = (\vec{P}_f)_{\text{system}}$$

$$(K \cdot E)_{\text{B/C}} = (K \cdot E)_{\text{A/C}}$$

$$e = 1$$

$$e = \frac{\vec{v}_2 - \vec{v}_1}{u_1 - u_2} = 1$$

Inelastic

$$\vec{P}_i = \vec{P}_f$$

$$(K \cdot E)_{\text{A/C}} < (K \cdot E)_{\text{B/C}}$$

$$0 < e < 1$$

Perfectly Inelastic

$$(\vec{P}_i)_{\text{system}} = (\vec{P}_f)_{\text{system}}$$

$$(K \cdot E)_{\text{A/C}} < (K \cdot E)_{\text{B/C}}$$

$$\Delta K \cdot E_{\text{loss}} = K \cdot E_i - K \cdot E_f$$

= max

$$e = 0$$

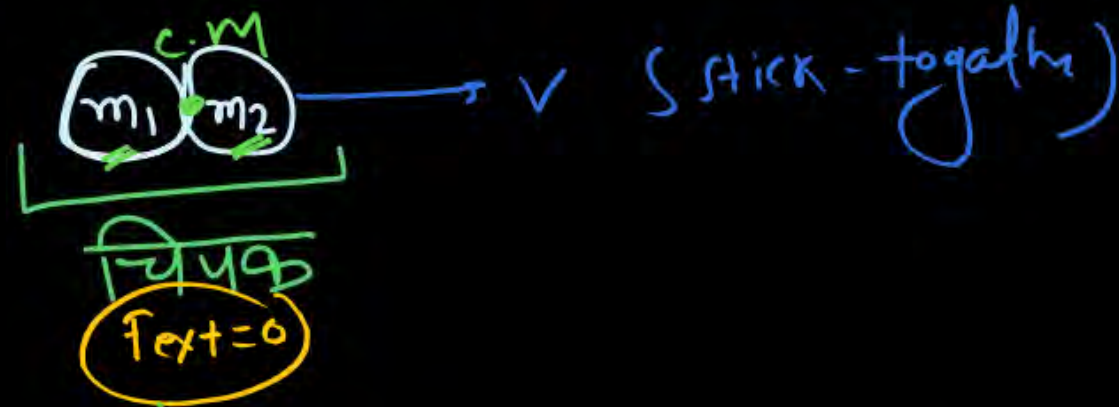
Stick

Statement True/False

- (1) In elastic collision momentum of object conserved $\rightarrow$ false
- (2) In every collision K.E of system conserved $\rightarrow$ false
- (3) For collision contact is necessary $\rightarrow$ false
- (4) Maximum loss in K.E in perfectly elastic collision $\rightarrow$ false
- (5) Value of e can be -ve $\rightarrow$ false
- (6) In elastic - collision K.E of system is conserved at all the time $\rightarrow$ false
- (7) Maxⁿ loss in K.E is in Perfectly Inelastic collision $\rightarrow$ True

Perfectly Inelastic collision (chipakne wala collision)

Permanently
(chipakne wala collision)



Solⁿ

$$e = 0 = \frac{\vec{v}_2 - \vec{v}_1}{(u_1 - u_2)}$$

$$m_1 \vec{u}_1 + m_2 \vec{u}_2 = (m_1 + m_2) \vec{v}$$

final velocity
Same for
Both

$$\vec{V}_2 - \vec{V}_1 = 0$$

$$\vec{V}_2 = \vec{V}_1 = v(\text{rel})$$

$$V = \frac{m_1 \vec{u}_1 + m_2 \vec{u}_2}{m_1 + m_2} \quad (\text{final velocity})$$

$$\vec{V}_{c.m} = \frac{m_1 \vec{u}_1 + m_2 \vec{u}_2}{m_1 + m_2}$$

~~##~~ Both

$$\Delta K.E_{\text{loss}} = \frac{1}{2} \frac{m_1 m_2}{(m_1 + m_2)} (\vec{u}_1 - \vec{u}_2)^2$$

$$\Delta K.E = K.E_f - K.E_i$$

$$= \frac{1}{2} (m_1 + m_2) v_{c.m.}^2 - \left(\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \right)$$

$$\Delta K.E = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_{rel})^2$$

Question



Two bodies A and B of same mass undergo completely inelastic one dimensional collision. The body A moves with velocity v_1 while body B is at rest before collision. The velocity of the system after collision is v_2 . The ratio $v_1 : v_2$ is NEET [2024]

1 1 : 2

2 2 : 1 ✓

3 4 : 1

4 1 : 4



$$mv_1 + m \times 0 = 2mv_2$$

$$\frac{mv_1}{2m} = v_2$$

$$v_2 = \frac{v_1}{2}$$
$$\boxed{2 = \frac{v_1}{v_2}}$$

$$4\text{kg} \rightarrow u_1 = 10\text{m/s}$$

$$6\text{kg} \rightarrow u_2 = 5\text{m/s}$$

if collision is Perfectly Inelastic, then find

- final common velocity
- loss in K.E.

$$4 \times 10 + 6 \times 5 = (4+6) V_c$$

$$40 + 30 = 10 V_c$$

$$V_1 = V_2 = V_c = \frac{70}{10} = 7\text{m/s}$$

$$\Delta K.E_{\text{loss}} = \frac{1}{2} \frac{4 \times 6}{(4+6)} (10-5)^2$$

$$= \frac{120}{2} \times 25 = 30\text{J}$$

MR* BOX

- question me sabse pahle Type of Collision decide Karo.

$$\Delta K.E_{\text{loss}} = K.E_f - K.E_i$$

$$= \frac{1}{2} (10) (7)^2 - \left(\frac{1}{2} 4 (10)^2 + \frac{1}{2} 6 (5)^2 \right)$$

$$= 49 \times 5 - (200 + 75)$$

$$= 245 - 275$$

$$= -30\text{J}$$

↑
loss



find final velocity if collision is :-

(i) $e = 0$ (perfectly Inelastic)

(Stick)

$$6 \times 20 + 4 \times 0 = (10)V$$

$$120 + 0 = 10V$$

$$V = 12 \text{ m/s} = V_1 = V_2$$

$$\Delta KE_{14} = \frac{1}{2} \times \frac{6 \times 4}{10} (20 - 0)^2 = \frac{12}{10} \times 400 = 480 \text{ J}$$

(ii) elastic ($e = 1$)

$$m_1 u_1 + m_2 u_2 = 6v_1 + 4v_2 \quad \text{--- (1)}$$

$$e = 1 = \frac{v_2 - v_1}{20 - 0}$$

$$6 \times 20 + 4 \times 0 = 6v_1 + 4v_2$$

$$120 = 6v_1 + 4v_2$$

$$30 = 1.5v_1 + v_2$$

$$24 = v_2$$

$$20 = v_2 - v_1$$

$$20 + v_1 = v_2 \quad \text{--- (11) Put in eq (1)}$$

$$6 \times 20 + 4 \times 0 = 6v_1 + 4(20 + v_1)$$

$$120 = 6v_1 + 80 + 4v_1$$

$$40 = 10v_1$$

$$\Delta K.E_{\text{loss}} = 0$$

$$v_1 = 4 \text{ m/s}$$

Question



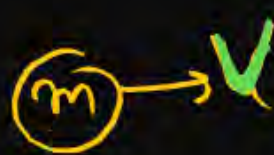
A moving block having mass m , collides with another stationary block having mass $4m$. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v , then the value of coefficient of restitution (e) will be **NEET-[2018]**

1 0.5

2 0.25

3 0.8

4 0.4



$$mv + 0 = 0 + 4mv_2$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\frac{v}{4} - 0}{v - 0} \quad v_2 = \frac{v}{4} \text{ --- (1)}$$

$$e = \frac{v}{4v} = \frac{1}{4}$$

Question



A bullet of mass m moving with a velocity u strikes a block of mass M at rest and gets embedded in the block. The loss of kinetic energy in the impact is

NEET

1 $\frac{1}{2}mMu^2$

2 $\frac{1}{2}(m+M)u^2$

3 $\frac{mMu^2}{2(m+M)}$

4 $\left(\frac{m+M}{2mM}\right)u^2$



$$\Delta K.E_{\text{loss}} = \frac{1}{2} \frac{mM}{m+M} u^2$$

Question



An object of mass 80 kg moving with velocity 2 ms^{-1} hit by collides with another object of mass 20 kg moving with velocity 4 ms^{-1} . Find the loss of energy assuming a perfectly inelastic collision

1 12 J *

2 24 J

3 30 J

4 32 J

$$\Delta K.E = \frac{1}{2} \frac{80 \times 20}{100} (4-2)^2 \checkmark$$

Question



A ball moving with velocity 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0.5, then their velocities (in m/s) after collision will be **[2010] NEE**

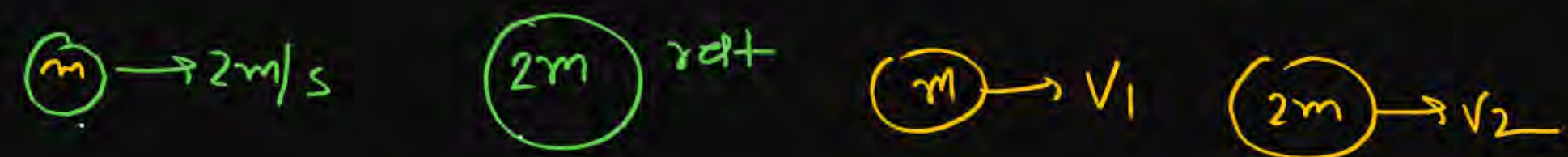
1 0, 1 ~~AB~~

2 ~~1, 1~~

3 ~~1, 0.5~~

4 0, 2

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$
$$0.5 = \frac{v_2 - v_1}{2 - 0}$$
$$1 = v_2 - v_1$$
$$v_2 = 1 + v_1$$



$e = 0.5$
Inelastic collision

$$2m + 2m \times 0 = mv_1 + 2mv_2$$

$$2 = v_1 + 2v_2 \quad \text{--- (1)}$$

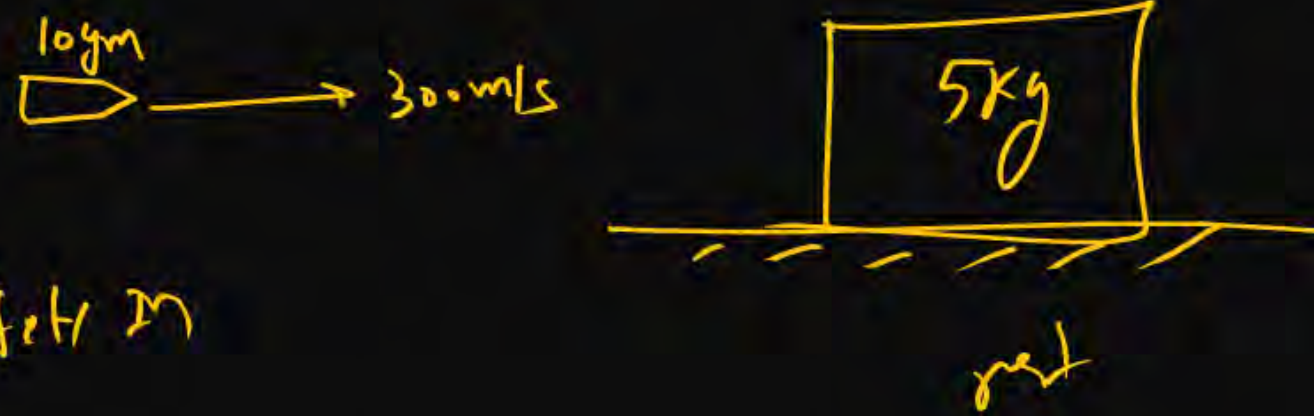
$$2 = v_1 + 2(1 + v_1)$$

$$2 = v_1 + 2 + 2v_1$$
$$2 - 2 = 3v_1 \quad 3v_1 = 0 \quad v_1 = 0$$

Question

A bullet weighing 10 g and moving with a velocity 300 m/s strikes a 5 kg block of ice and drop dead. The ice block is kept on smooth surface. The speed of the block after the collision is

- 1 6 cm/s
- 2 60 cm/s
- 3 6 m/s
- 4 0.6 cm/s



$$\frac{10}{1000} \times 300 + 0 = 5v$$

$$\frac{3}{5} = v$$

$$v = 0.6 \text{ m/s}$$

Question



A body of mass 10 kg moving with speed of 3 ms⁻¹ collides with another stationary body of mass 5 kg. As a result, the two bodies stick together. The KE of composite mass will be

- 1 30 J ✓
- 2 60 J
- 3 90 J
- 4 120 J

$$10 \times 3 + 5 \times 0 = 15 \checkmark$$
$$\frac{15^2}{2} = 112.5 \checkmark$$

$$K.E_{sys} = \frac{1}{2} (15) (2)^2 = 30$$

Question

NEET 2026



A mass m moving horizontally (along the x -axis) with velocity v collides and sticks to a mass of $3m$ moving vertically upward (along the y -axis) with velocity $2v$. The final velocity of the combination is

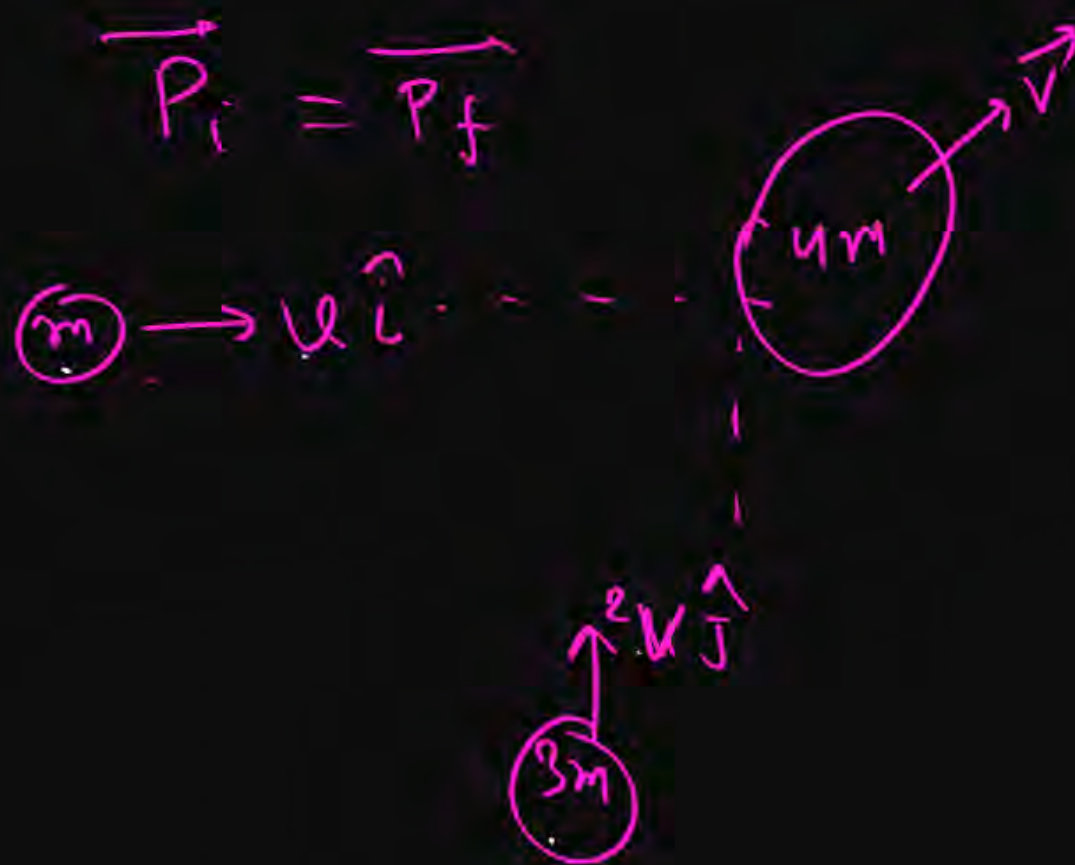
[Mains 2011]

1 $\frac{3}{2}v\hat{i} + \frac{1}{4}v\hat{j}$

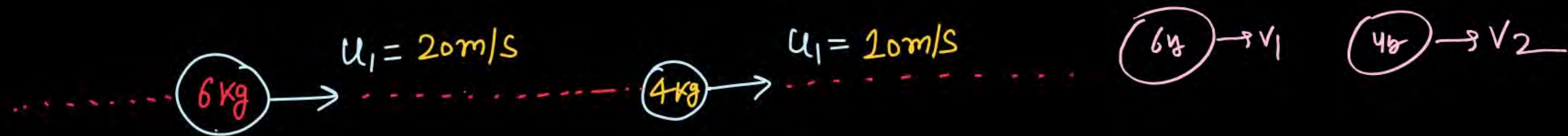
2 $\frac{1}{4}v\hat{i} + \frac{3}{2}v\hat{j}$

3 $\frac{1}{3}v\hat{i} + \frac{2}{3}v\hat{j}$

4 $\frac{2}{3}v\hat{i} + \frac{1}{3}v\hat{j}$



$$mv\hat{i} + 3m2v\hat{j} = 4mV$$
$$\frac{v}{4}\hat{i} + \frac{3}{2}v\hat{j} = V$$



find final velocity if collision is :-

(i) $e=0$ perfectly Inelastic

$$6 \times 20 + 4 \times 10 = 10(v)$$

$$12 + 4 = v$$

$$\boxed{v = 16\text{m/s}} = v_1 = v_2$$

(ii) elastic ($e=1$)

$$20 \times 6 + 4 \times 10 = 6v_1 + 4v_2$$

$$1 = \frac{v_2 - v_1}{20 - 10}$$

$$10 = v_2 - v_1$$

$$\boxed{v_2 = 10 + v_1 - \textcircled{1}}$$

$$120 + 40 = 6v_1 + 4(10 + v_1)$$

$$160 = \underline{6v_1} + 40 + \underline{4v_1}$$

$$120 = 10v_1$$

$$v_1 = 12\text{m/s}$$

$$\boxed{v_2 = 10 + v_1 = 22\text{m/s}}$$

(iii) inelastic $e = 1/2$

$$20 \times 6 + 10 \times 4 = 20v_1 + 10v_2 \quad \text{--- (1)}$$

$$e = \frac{v_2 - v_1}{20 - 10} = \frac{1}{2}$$

$$\frac{v_2 - v_1}{\frac{10}{5}} = \frac{1}{2}$$

$$v_2 - v_1 = 5$$

$$\boxed{v_2 = 5 + v_1}$$

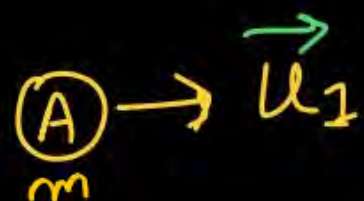
$$160 = 20v_1 + 10(5 + v_1)$$

$$160 = \underline{20v_1} + 50 + \underline{5v_1}$$

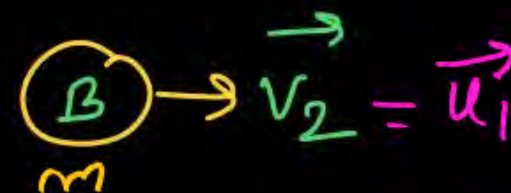
$$110 = 25v_1$$

$$v_1 = \frac{110}{25} \checkmark$$

Elastic collision of two object of same mass :-



→
A/C



velocity will exchange

$$P_i = P_f$$

$$m\vec{u}_1 + m\vec{u}_2 = m\vec{v}_1 + m\vec{v}_2$$

$$\vec{u}_1 + \vec{u}_2 = \vec{v}_1 + \vec{v}_2 \quad \text{Put } v_2 \text{ from eqn (1)}$$

$$\vec{u}_1 + \vec{u}_2 = \vec{v}_1 + u_1 - u_2 + v_1$$

$$e = 1 = \frac{v_2 - v_1}{u_1 - u_2}$$

$$u_1 - u_2 = v_2 - v_1$$

$$v_2 = u_1 - u_2 + v_1 \quad \text{--- (11)}$$

$$u_1 + u_2 - u_1 + u_2 = 2v_1$$

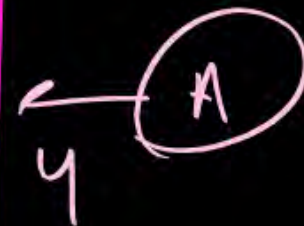
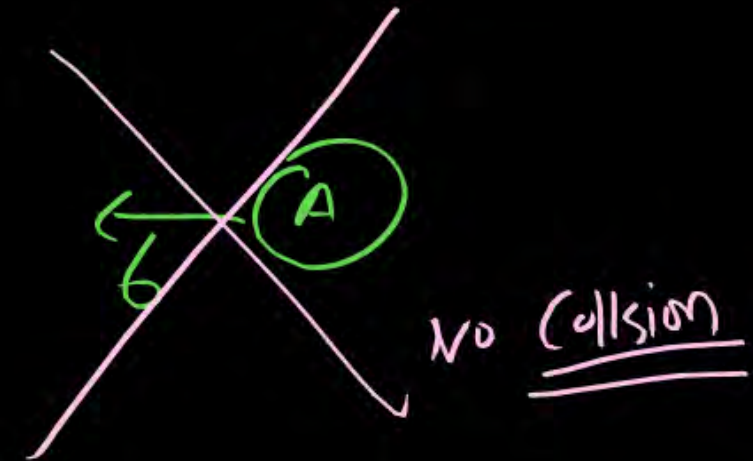
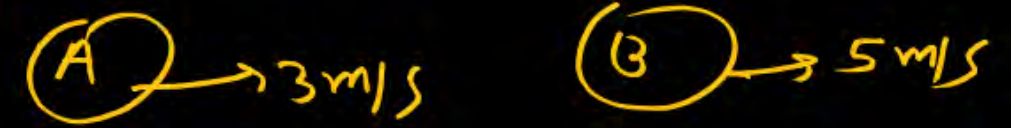
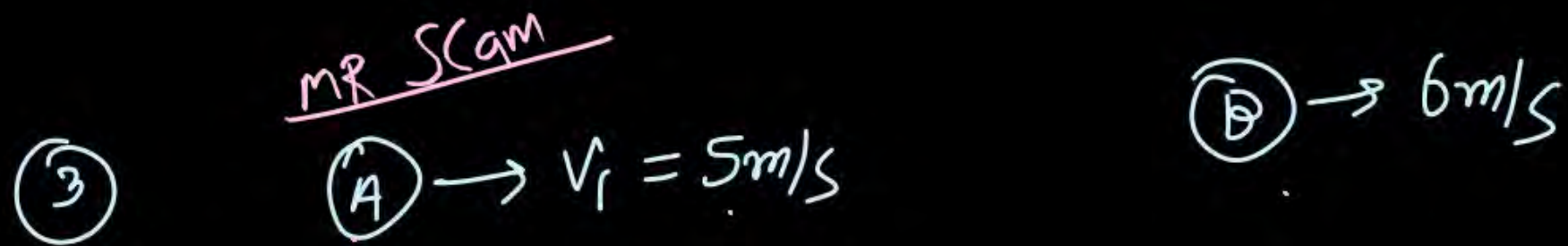
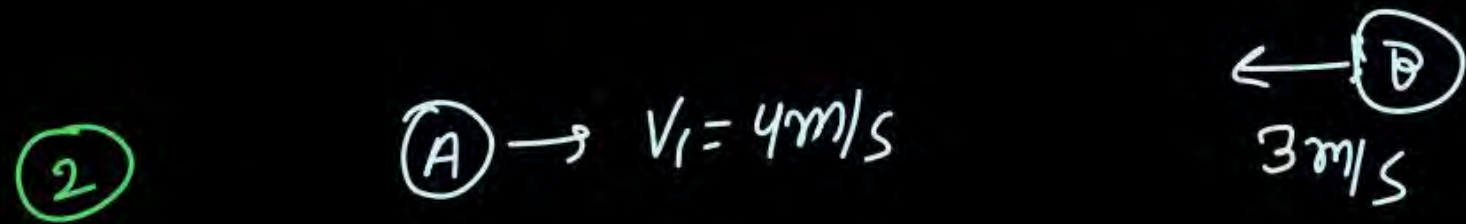
$$u_2 = v_1$$

$$\vec{v}_1 = \vec{u}_2$$

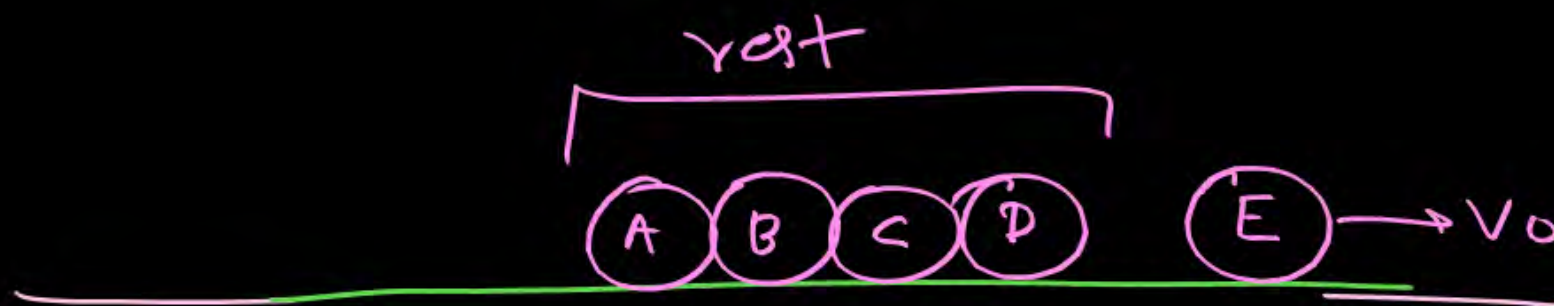
↑ final velocity of 1st object

(initial velocity of 2nd object)

Elastic collision of same mass.



all have same mass & elastic collision.



Question



Two ivory balls are placed together at rest. A third identical ball moving with velocity u in line of first two balls; as shown in figure collide head on elastically then



- 1 Third ball comes to rest with second ball while first ball moves with speed u
- 2 Third ball comes to rest and other ~~two~~ move together with speed $u/2$
- 3 All ~~three~~ ball move together with speed $u/3$
- 4 All three ball move in such a manner each makes angle 120° to each other

Question



Two identical balls A and B having velocities of 0.5 m s^{-1} and -0.3 m s^{-1} respectively collide elastically in one dimension. The velocities of B and A after the collision respectively will be
[NEET-II 2016, 1994, 1991]

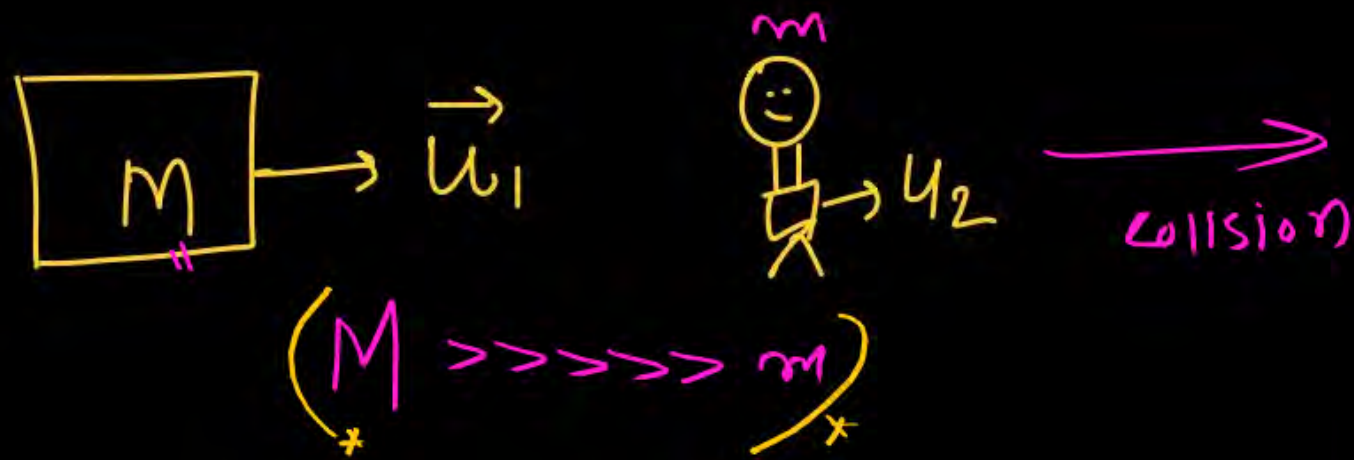
1 -0.5 m s^{-1} and 0.3 m s^{-1}

2 0.5 m s^{-1} and -0.3 m s^{-1}

3 -0.3 m s^{-1} and 0.5 m s^{-1}

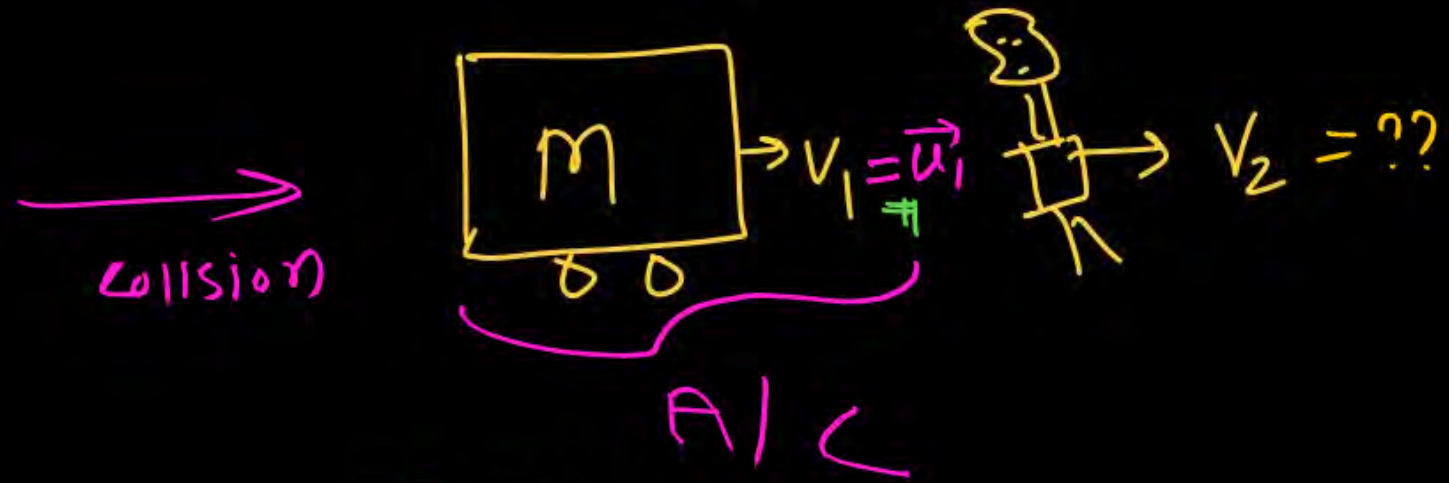
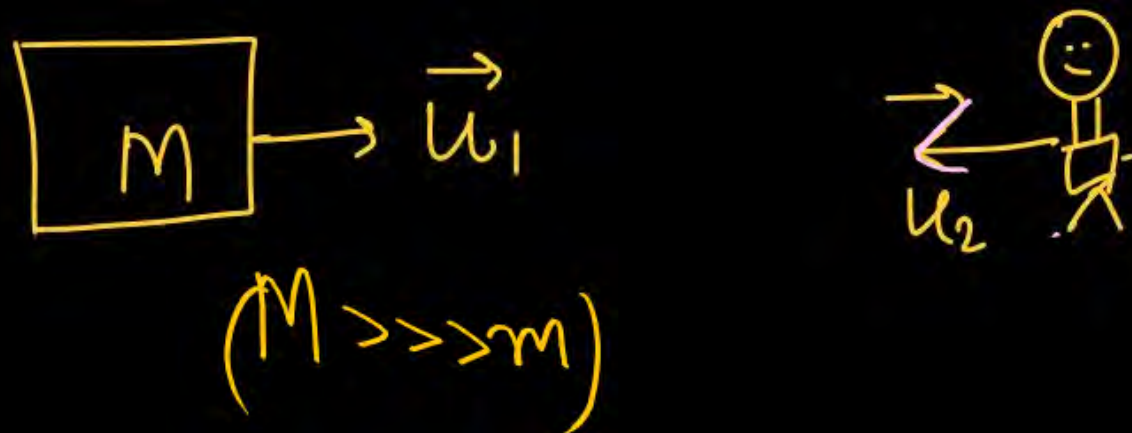
4 0.3 m s^{-1} and 0.5 m s^{-1}

✓ Elastic collision of two object, In which mass of one is very much greater than other ($m \gg m$)



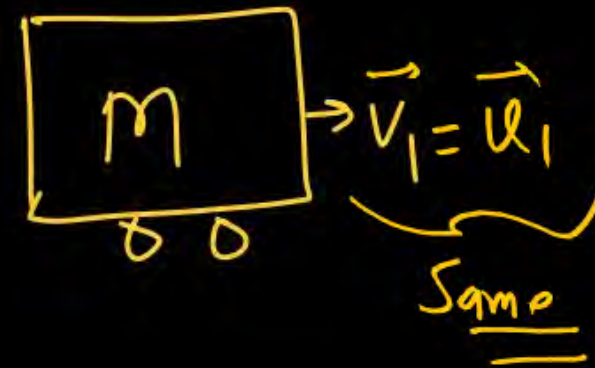
$$u_1 - u_2 = \vec{v}_2 - u_1$$

$$\vec{v}_2 = 2\vec{u}_1 - \vec{u}_2$$



$$e = \frac{\vec{v}_2 - \vec{v}_1}{\vec{u}_1 - \vec{u}_2}$$

$$1 = \frac{v_2 - u_1}{u_1 - u_2}$$



$$\vec{v}_2 = 2\vec{u}_1 + \vec{u}_2$$

Question



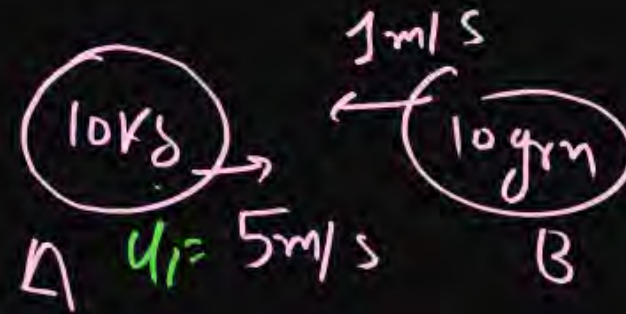
Two steel balls A and B of mass 10 kg and 10 g rolls towards each other with 5 m/s and 1 m/s respectively on a smooth floor. After collision; with what speed B moves [perfectly elastic collision]?

1 8 m/s

2 10 m/s

3 11 m/s ✓✓

4 Zero



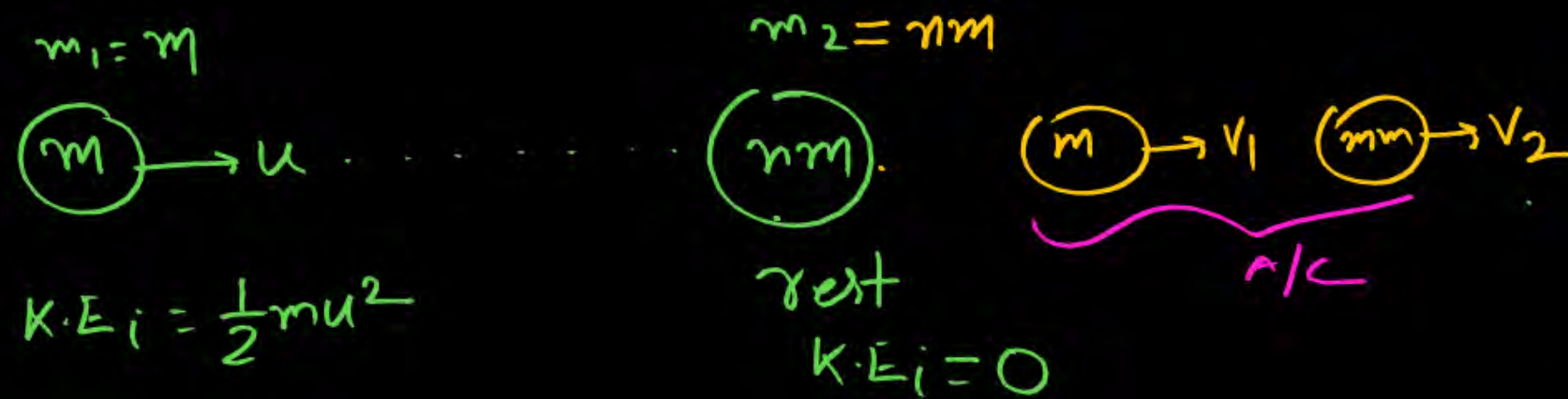
$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$1 = \frac{v_2 - 5}{5 - (-1)}$$

$$6 = v_2 - 5$$

$$v_2 = 6 + 5 = 11 \text{ m/s}$$

Object of mass $m_1 = m$ moving with velocity u collide elastically* with other object of mass $m_2 = nm$ which is at rest. then find fraction of transferred K.E to m_2 from m_1 & retained by m_1



$$\textcircled{1} \quad \frac{m_2}{m_1} = \frac{nm}{m}$$

$$n = \frac{m_2}{m_1}$$

Soln

$$\cancel{mu} + nm \times 0 = \cancel{mv_1} + nm v_2 \quad \text{---} \textcircled{1}$$

$$u = v_1 + nv_2 \quad \text{---} \textcircled{1}$$

$$u = v_2 - u + nv_2$$

$$2u = v_2(1+n)$$

$$\underline{v_2 = \frac{2u}{(1+n)}}$$

$$1 = \frac{v_2 - v_1}{u - 0}$$

$$u = v_2 - v_1$$

$$\boxed{v_1 = v_2 - u \quad \text{---} \textcircled{1}}$$

$$\textcircled{2} \quad K.E_{\text{loss system}} = 0$$

$$\textcircled{3} \quad \frac{K.E_2}{K.E_{\text{init of 1st object}}} = ??$$

K.E of 2nd object

$$\begin{aligned} K.E_{2nd\ object} &= \frac{1}{2} m m v_2^2 \\ &= \frac{1}{2} m m \left(\frac{2u}{1+n} \right)^2 \end{aligned}$$

$$\# \frac{K.E_{Transferred}}{K.E_{initial\ of\ m_1}} = \frac{\cancel{\frac{1}{2}} m m \left(\frac{2u}{1+n} \right)^2}{\cancel{\frac{1}{2}} m u^2} = \frac{n \cancel{4} u^2}{(1+n)^2 \cancel{u^2}} = \frac{4n}{(1+n)^2}$$

$$\textcircled{\#} \frac{K.E_{Trans}}{K.E_i} = \frac{4n}{(1+n)^2}$$

$$\frac{K.E_{retained\ by\ m_1}}{K.E_{initial\ by\ m_1}} = \left(\frac{1-n}{1+n} \right)^2$$

Question



A neutron makes a head on elastic collision with a stationary deuteron. The fractional energy loss of the neutron in the collision is:

1 $16/82$

2 $8/9$ ✓

3 $8/27$

4 $2/3$



$$\text{frac of } Tm \text{ K.E} = \frac{4m}{(1+m)^2} = \frac{4 \times 2}{(1+2)^2} = \frac{8}{9}$$

Question



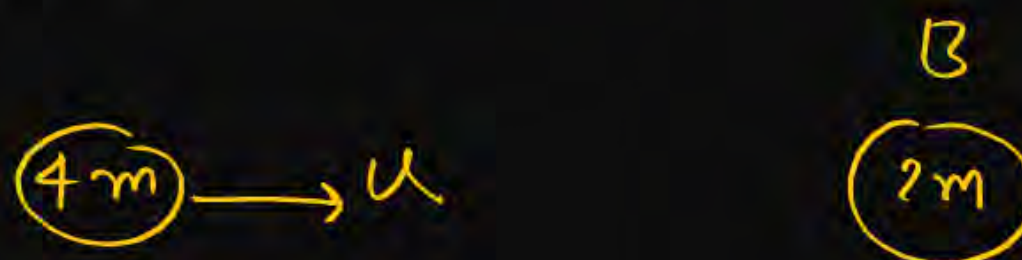
Body A of mass $4m$ moving with speed u collides with another body B of mass $2m$, at rest. The collision is head on and elastic in nature. After the collision the fraction of energy lost by the colliding body A is **[2023]**

1 $5/9$

2 $1/9$

3 $8/9$

4 $4/9$



$$Q = \frac{4m}{(1+n)^2} = \frac{4 \times 2}{(1+2)^2} = \frac{8}{9}$$

NEET-2019



fraction of transferred K.E to α -particle. if collision is elastic

$$\frac{4 \times 4}{(4+1)^2} = \frac{16}{25} \checkmark$$

Question

A bullet of mass m hits a block of mass M elastically. The transfer of energy is the maximum, when: **[Manipur NEET 2023]**

1

$$M = m$$

2

$$M = 2m$$

3

$$M \ll m$$

4

$$M \gg m$$



$$\frac{4m}{(1+m)^2} = 1$$

$$4m = (1+m)^2$$

$$\frac{m_2}{m_1} = n = 1$$

$$\text{sf } \underline{\underline{n=1}} \quad \underline{\underline{4 \times 1 = 4}} \quad \bigg| \quad \underline{\underline{(1+1)^2 = 4}}$$

Question



A ^{m_p} neutron collides head-on and elastically with an atom of mass number A which is initially at rest. The fraction of kinetic energy retained by neutron is

AI PMT-2013

~~1~~ $\left(\frac{A}{A+1}\right)^2$

2 $\left(\frac{A-1}{A+1}\right)^2$

~~3~~ $\left(\frac{A-1}{A}\right)^2$

~~4~~ $\left(\frac{A+1}{A-1}\right)^2$

$$= 1 - \frac{4n}{(1+n)^2} = \left(\frac{1-n}{1+n}\right)^2$$

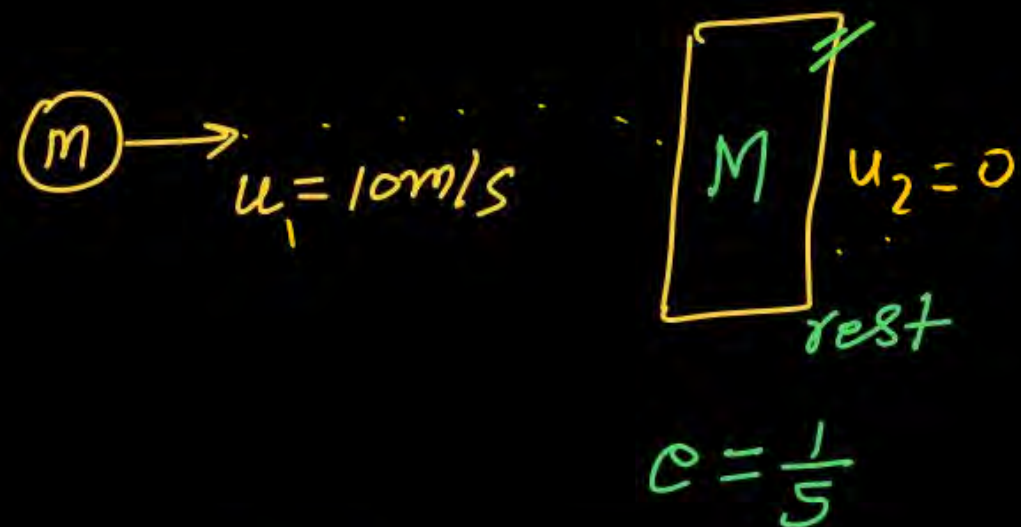
$$= \left(\frac{1-A}{1+A}\right)^2$$

~~$m_2 = m_p A$~~
 ~~$m_1 = m_p$~~

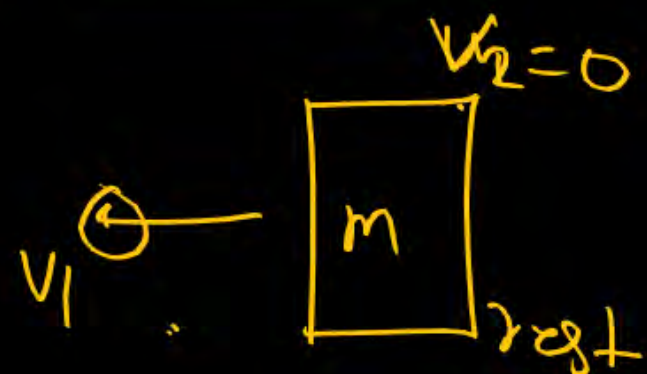
$n = \frac{m_2}{m_1} = A \checkmark$

find final velocity of each object if $(M \gg m)$

①



A/c



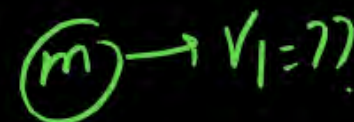
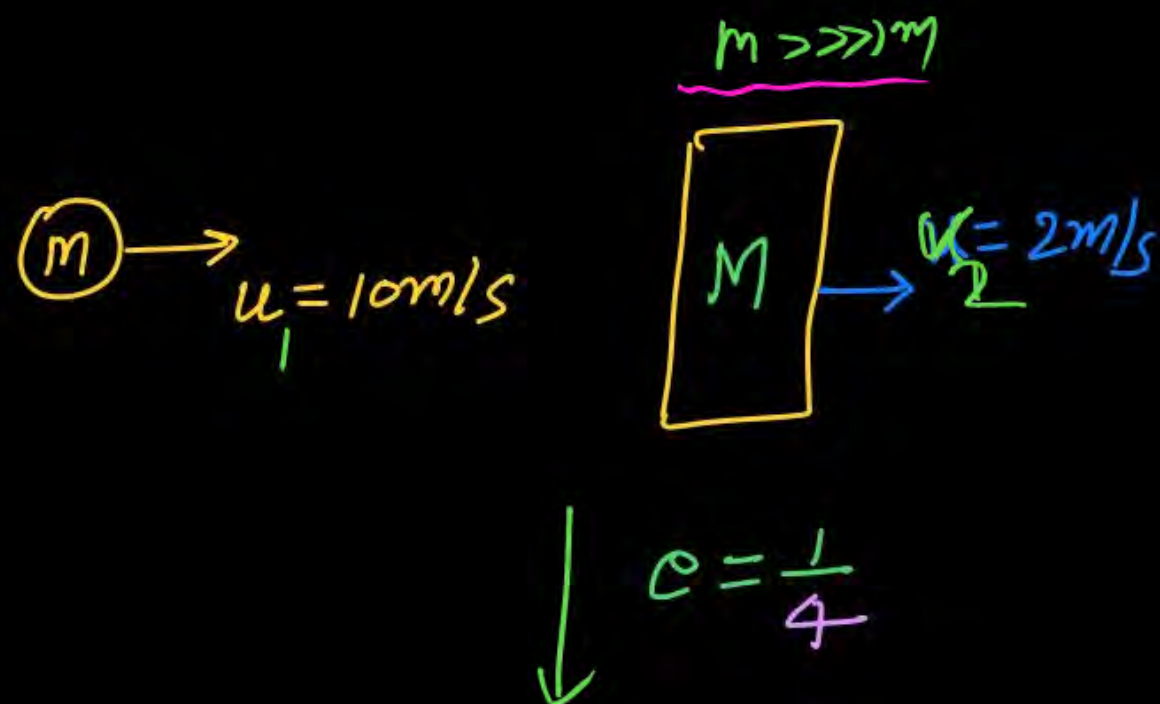
$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\frac{1}{5} = \frac{0 - v_1}{10 - 0}$$

$$\frac{1}{5} = -v_1$$

$$\boxed{v_1 = -2}$$

②



$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\frac{1}{4} = \frac{2 - v_1}{10 - 2}$$

$$\frac{1}{4} \times 8 = 2 - v_1$$

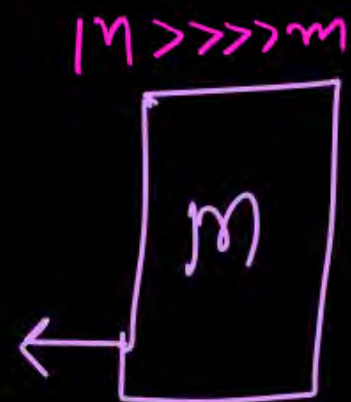
$$2 = 2 - v_1$$

$$v_1 = 2 - 2 = 0$$

0 *

③

$m \rightarrow u_1 = 4 \text{ m/s}$

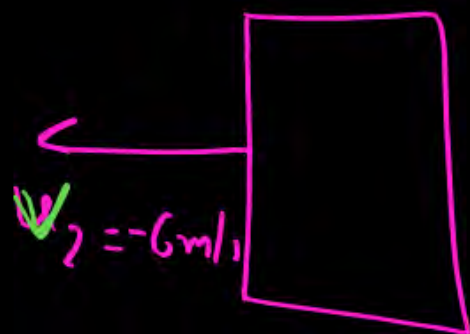


$u_2 = -6 \text{ m/s}$

$e = \frac{1}{2}$

A/C

$m \rightarrow v_1$

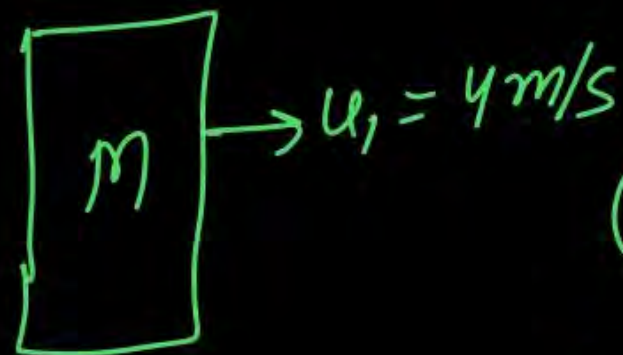


$e = \frac{v_2 - v_1}{u_1 - u_2}$

$\frac{1}{2} = \frac{-6 - v_1}{4 - (-6)}$

$\frac{10}{2} = -6 - v_1$

④



$m \rightarrow u_2 = 2 \text{ m/s}$

$e = \frac{1}{2}$

THANK
YOU

MISSION

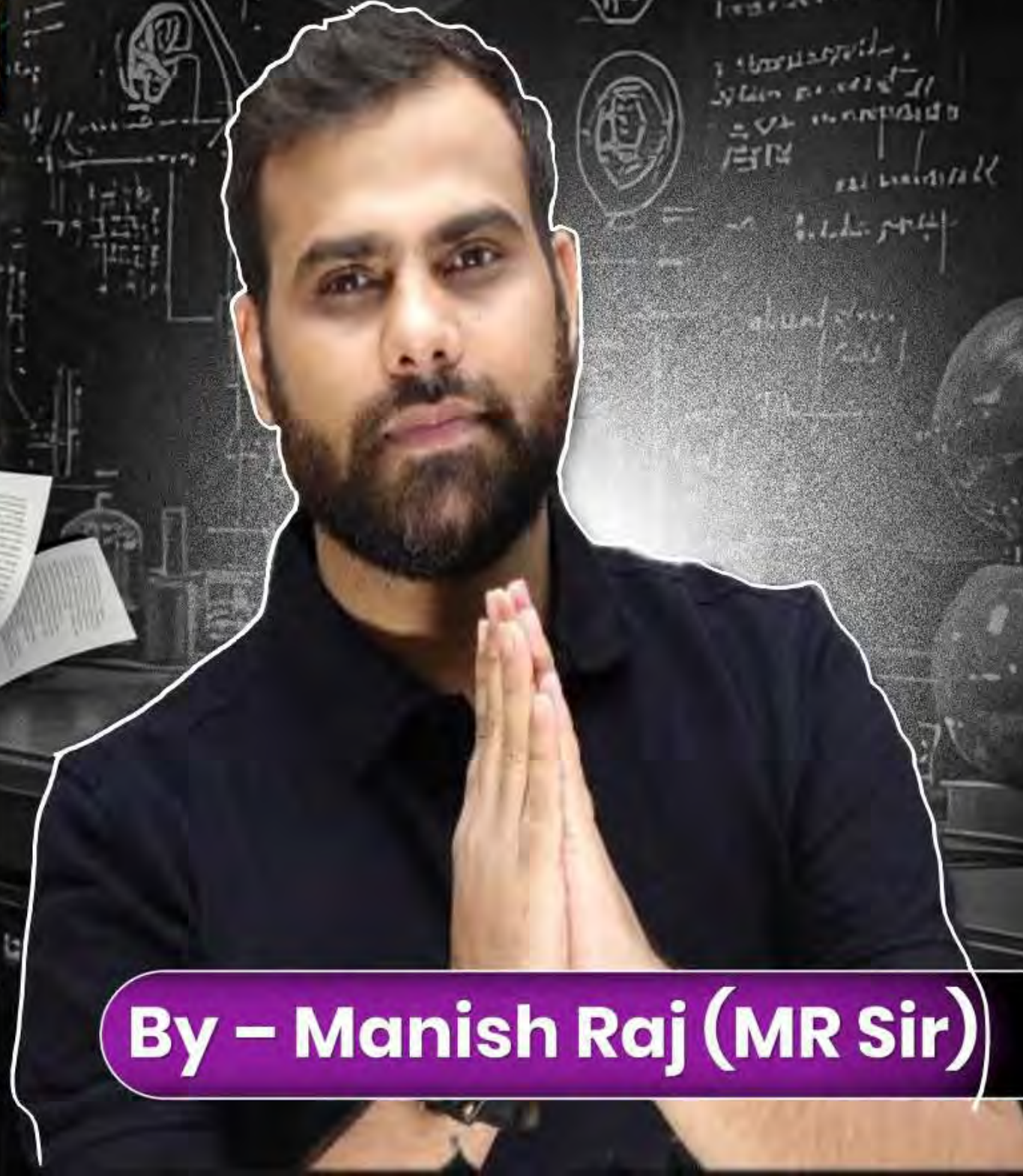
100

Centre of Mass and System of Particles

Physics

Lecture - 04

By – Manish Raj (MR Sir)

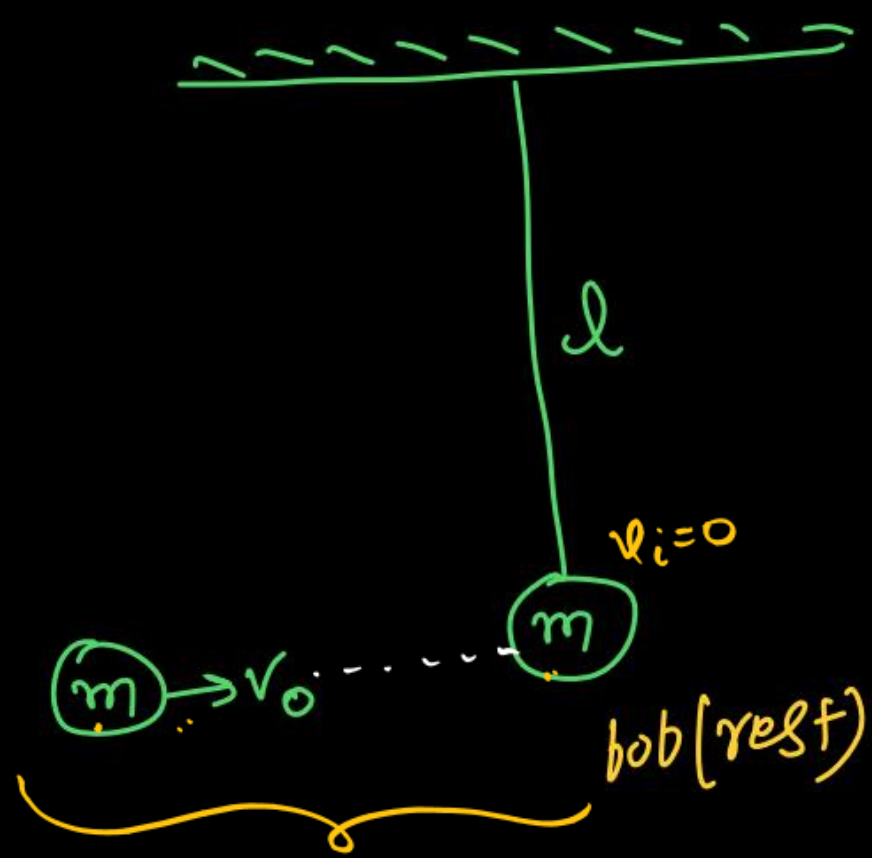


Today's Goal

1. Type of collision ✓
2. $P_i = P_f$ ✓
3. Koi specified case to Nahi

Oblique Collision
Man Plank System

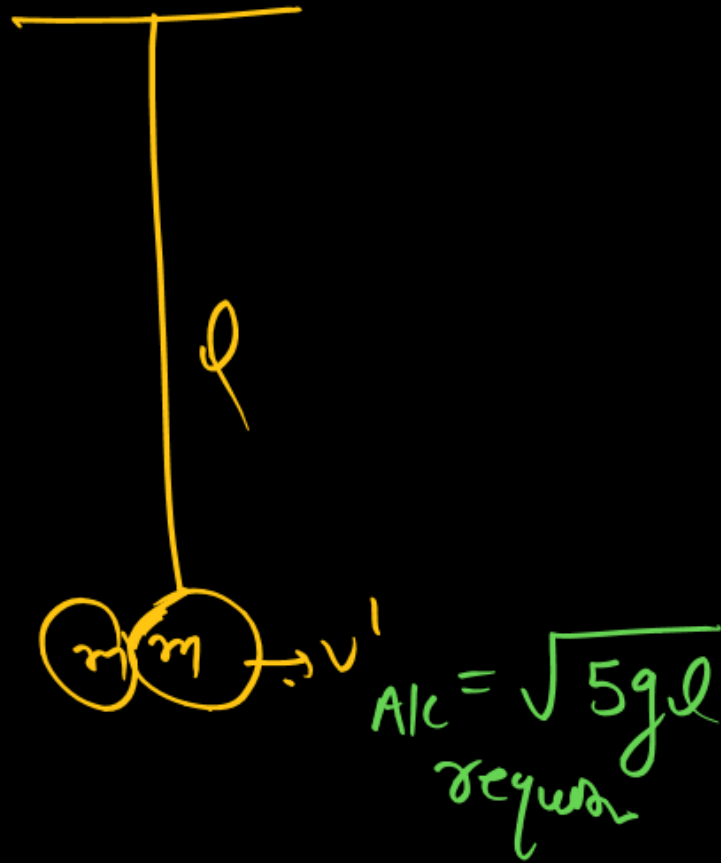
if collision is perfectly inelastic then find V_0 so that bob complete vertical circular motion.



$$p_i = mv_0 + 0 = 2mv'$$

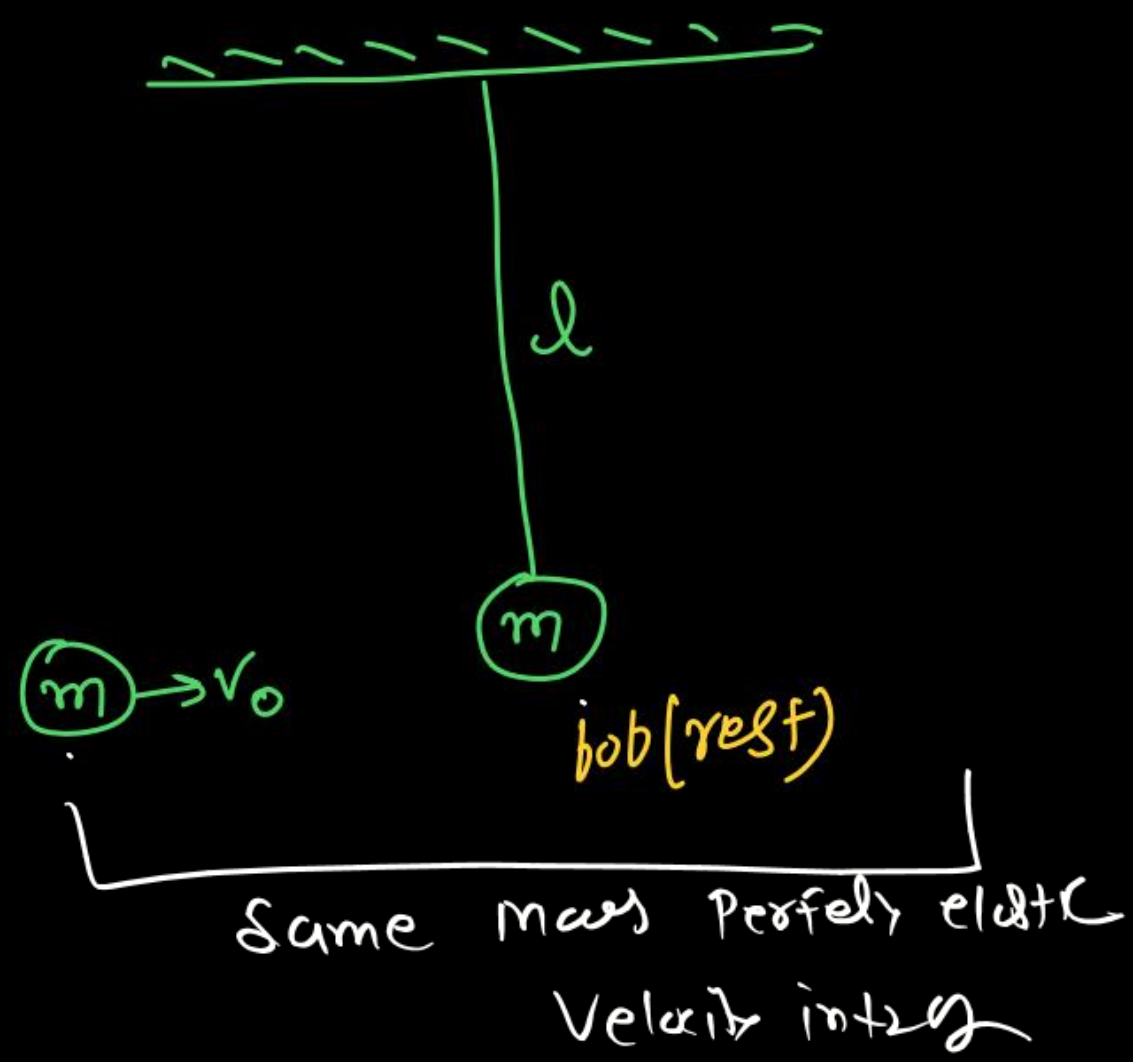
$$v' = \frac{v_0}{2}$$

gain

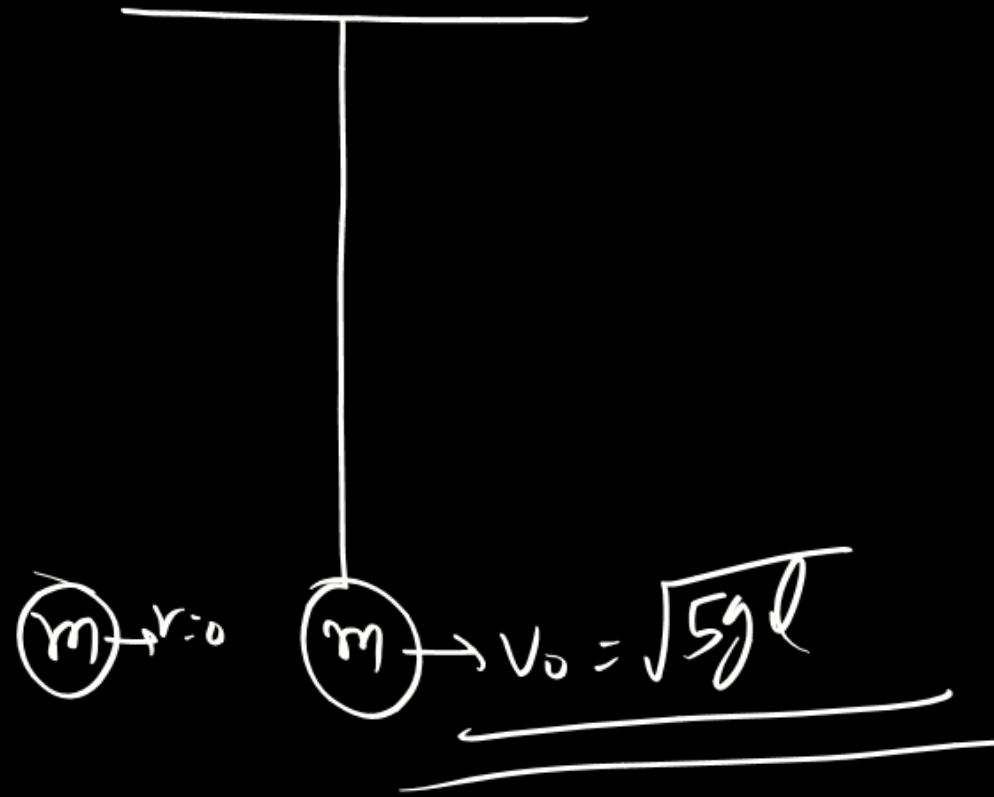


$$\frac{v_0}{2} = \sqrt{5gl}$$

$$v_0 = 2\sqrt{5gl} = \sqrt{20gl}$$

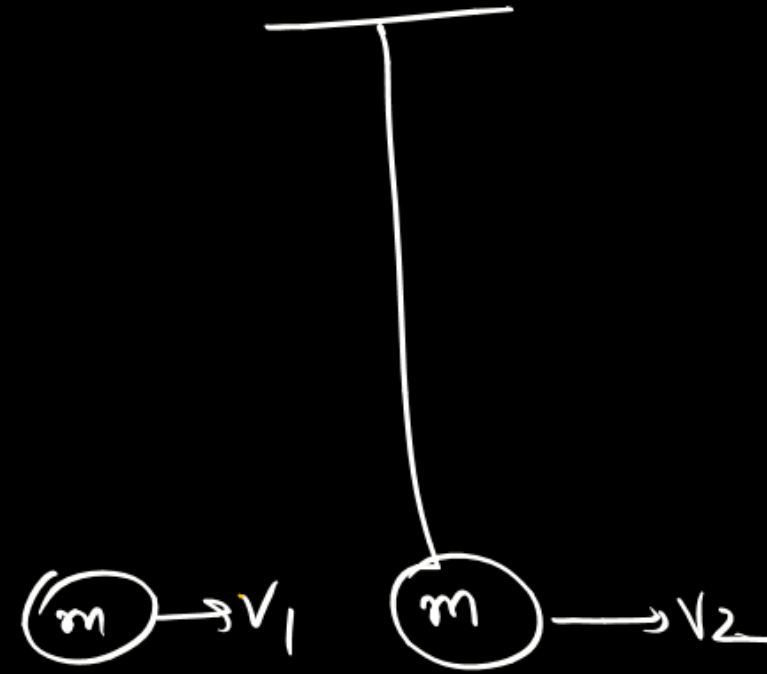
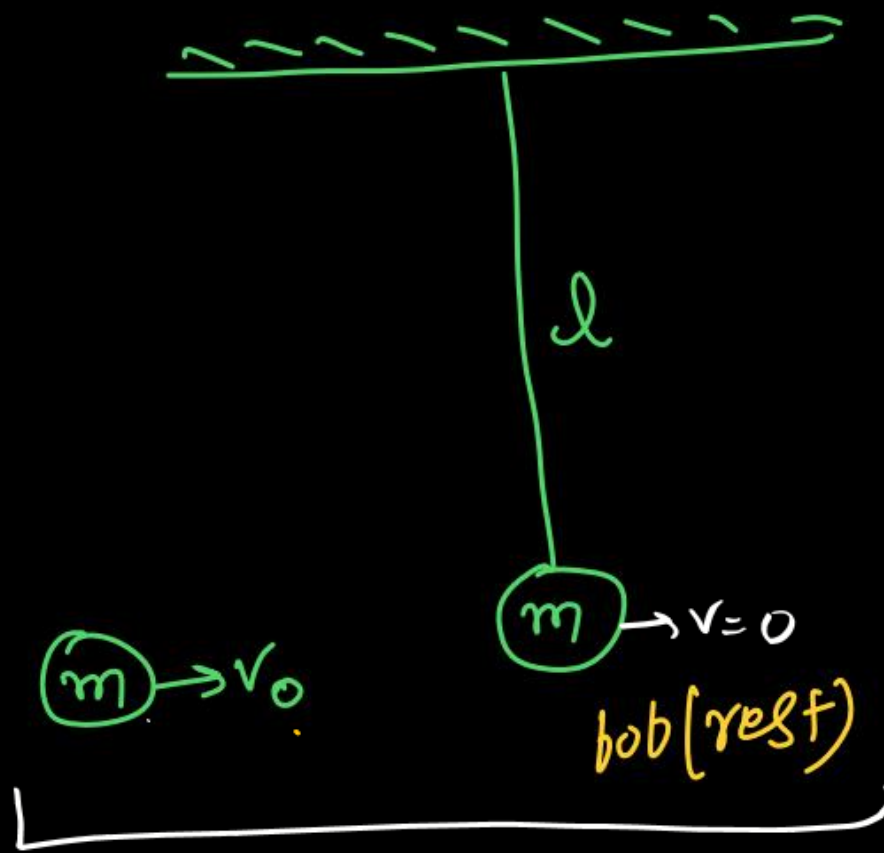


if collision is perfectly elastic then find v_0 so that bob complete vertical circular motion.



if collision is perfectly in elastic then find v_0 so that bob complete vertical circular motion.

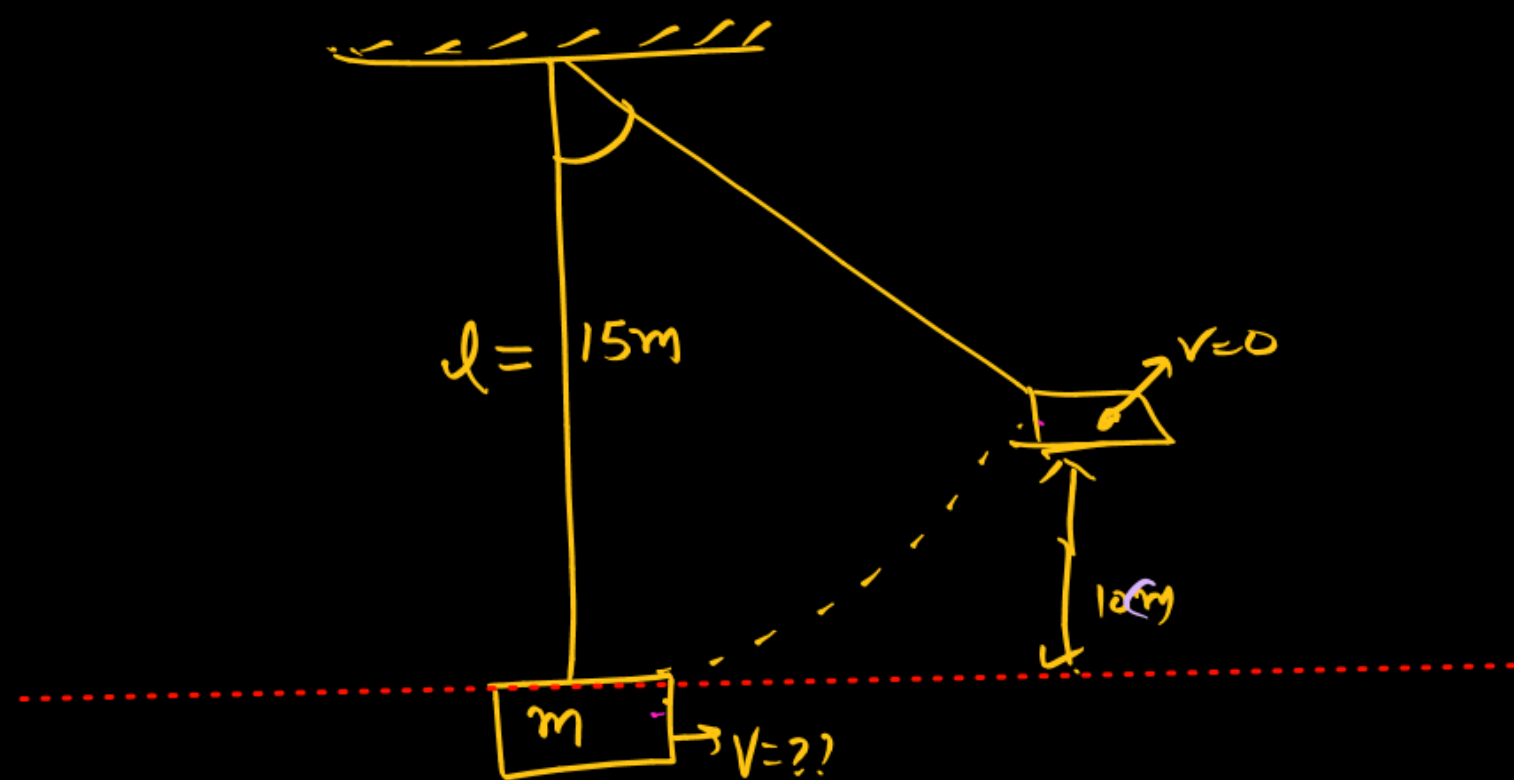
#



$$mv_0 + 0 = mv_1 + mv_2 \quad \text{--- (1)}$$

$$v_0 = v_1 + v_2 \quad \text{--- (1)}$$

$$e = \frac{1}{2} = \frac{v_2 - v_1}{v_0} \quad \text{--- (2)}$$



$$V = \sqrt{2g h_{\text{rem}}}$$

$$V = \sqrt{2 \times 10 \times \frac{10}{100}}$$

$$= \sqrt{2} \text{ m/s}$$

Velocity given so that it
rise to height 10 cm from bottom level.

Question



A bullet of mass 10 g moving horizontally with a velocity of 400 m/s strikes a wooden block of mass 2 kg which is suspended by a light inextensible string of length 5 m. As a result, the center of gravity of the block is found to rise a vertical distance of 10 cm. The speed of the bullet after it emerges out horizontally from the block will be:

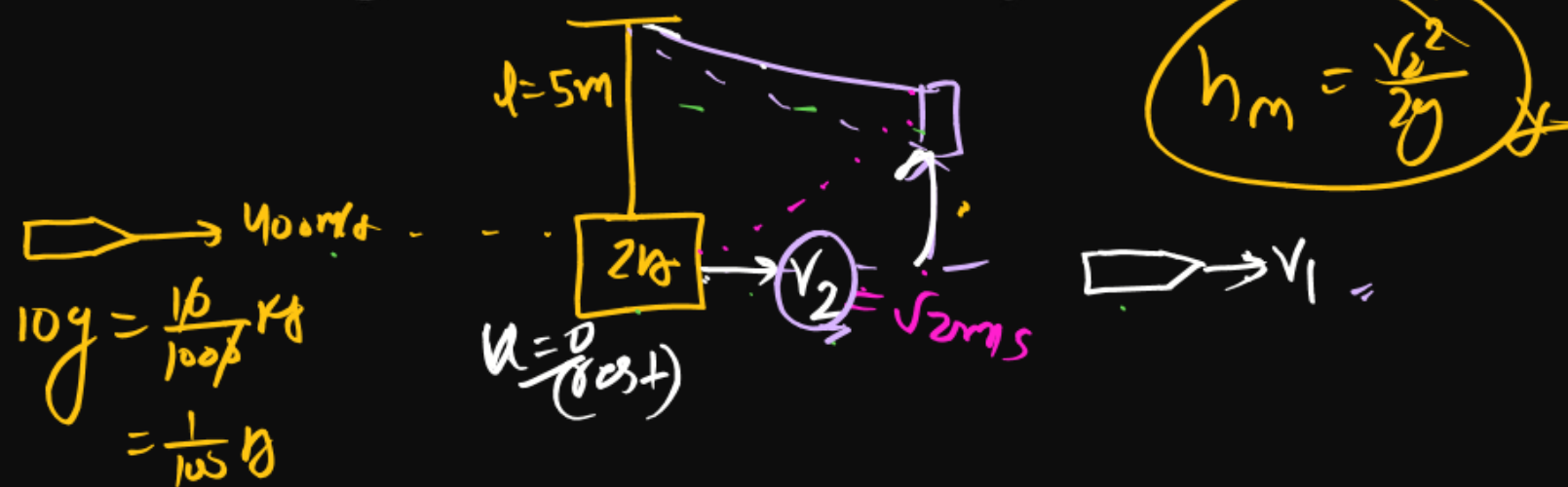
[NEET-2016]

1 ✓ 120 m/s

2 ✗ 160 m/s

3 ✗ 100 m/s

4 ✗ 80 m/s



$$\frac{1}{100} \times 400 + 0 = 2 \times v_2 + \frac{1}{100} v_1$$

$$4 - 2 \cdot 2.81 = \frac{v_1}{100}$$

$$1.19 \times 100 = v_1$$

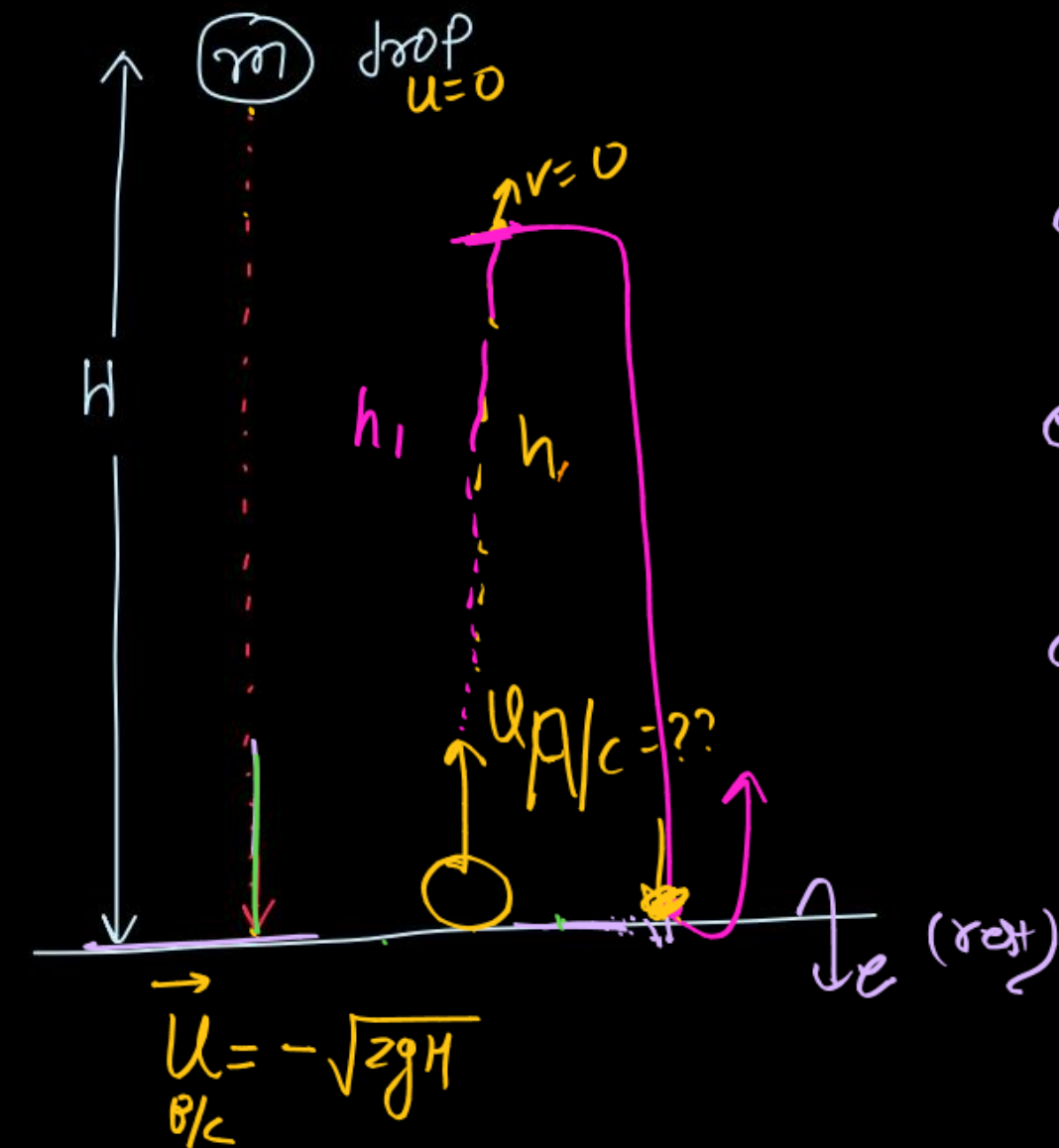
$$v_1 = 119 \text{ m/s}$$

$$4 = 2 \cdot 1.41 + \frac{v_1}{100}$$

$$4 - 2 \times 1.41 = \frac{v_1}{100}$$

Ball - earth collision :-

$P(\text{Ball} + \text{earth})$ conserved ✓



$$e = \frac{V_{sep}}{V_{app}}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$e = \frac{0 + V_{A/C}}{+\sqrt{2gH} - 0}$$

$$V_{A/C} = e \sqrt{2gH}$$

$$V_{2^{nd} A/C} = e^2 \sqrt{2gH}$$

$$V_{3^{rd} A/C} = e^3 \sqrt{2gH}$$

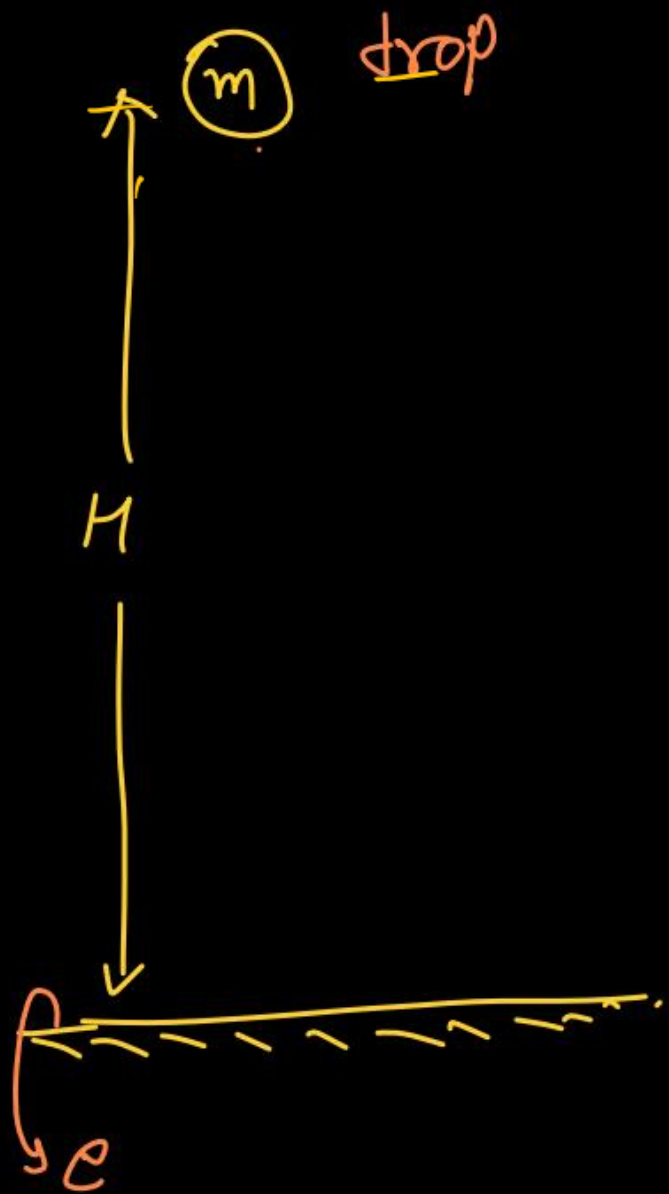
$$\text{After } n\text{-collision} = e^n \sqrt{2gH}$$

$$h_{A/C} = \frac{V_{A/C}^2}{2g} = \frac{e^2 \times 2gH}{2g} = e^2 H$$

$$h_{A/2^{nd} coll} = e^4 H$$

$$h_{A/3^{rd} coll} = e^6 H$$

$$h_{\text{After } n^{\text{th}} \text{ collision}} = e^{2n} H$$



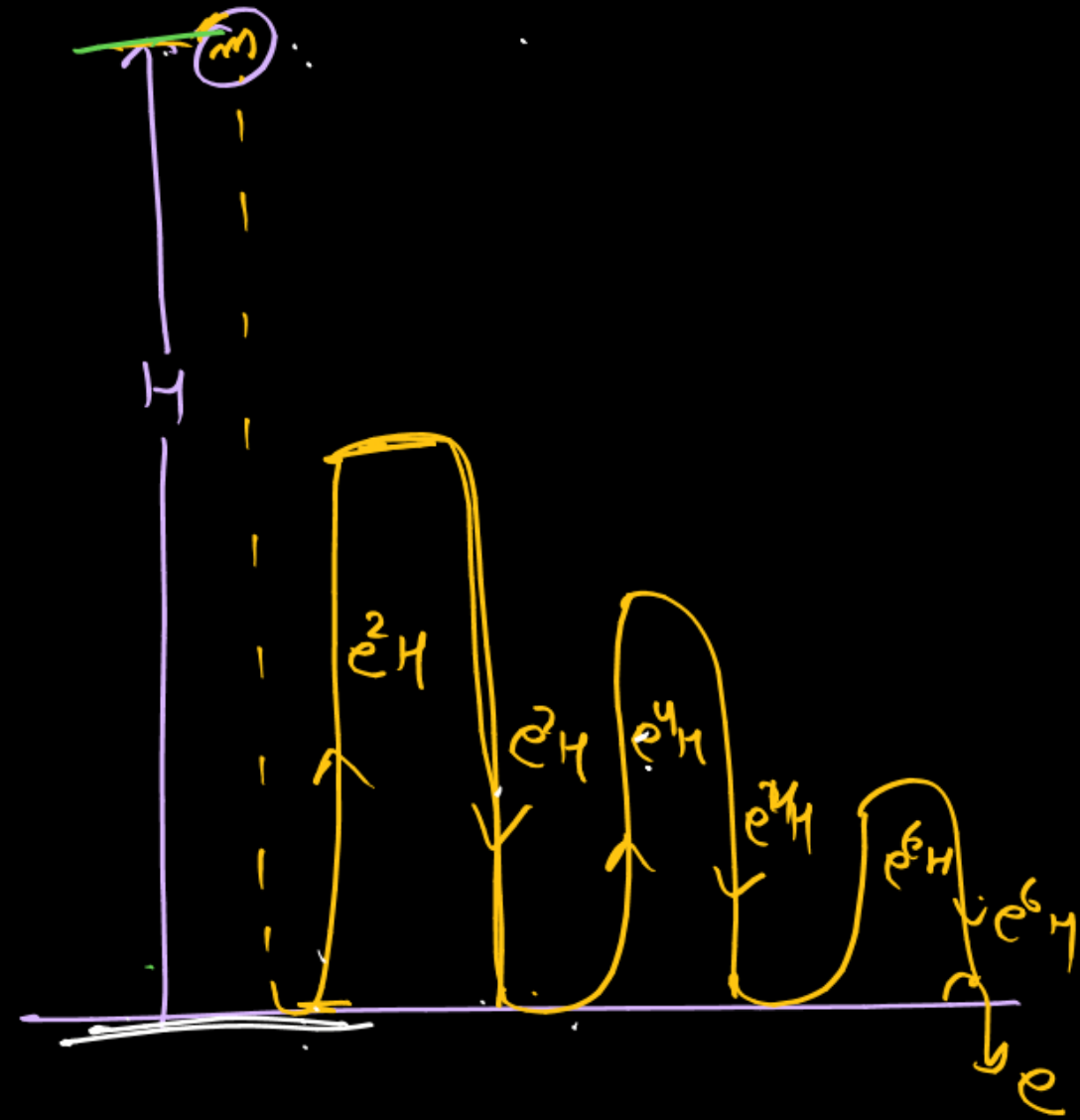
$$h_{A/C} = e^2 h_{B/C}$$

$$e = \sqrt{\frac{h_{A/C}}{h_{B/C}}}$$

$$e = \sqrt{\frac{H_{\text{max}}}{H_{\text{initial}}}} \checkmark$$

	After 1 st Collision	After 2 nd collision	After n^{th} Collision
velocity	$e \sqrt{2gh}$	$e^2 \sqrt{2gh}$	$e^n \sqrt{2gh}$
max. Height	$e^2 h$	$e^4 h$	$e^{2n} (H)$
Time of upward motion After n^{th}	$T = \frac{v_{A/C}}{g}$	X	X

object falls from height H on ground (e), find total distance travelled by body and time taken before comes to at rest. (NEET)



$$\text{Total distance} = H + 2e^2 H + 2e^4 H + 2e^6 H + 2e^8 H + \dots$$

$$= H \left(1 + \underbrace{2e^2 + 2e^4 + 2e^6 + 2e^8 + \dots}_{\text{G.P.}} \right)$$

$$= H \left(1 + \frac{2e^2}{1-e^2} \right) \quad \text{C.R.} = e^2$$

$$= H \left(\frac{1-e^2+2e^2}{1-e^2} \right) = H \left(\frac{1+e^2}{1-e^2} \right)$$

③ $e=0$
 $\text{dist} = H$

① total dist $> H$

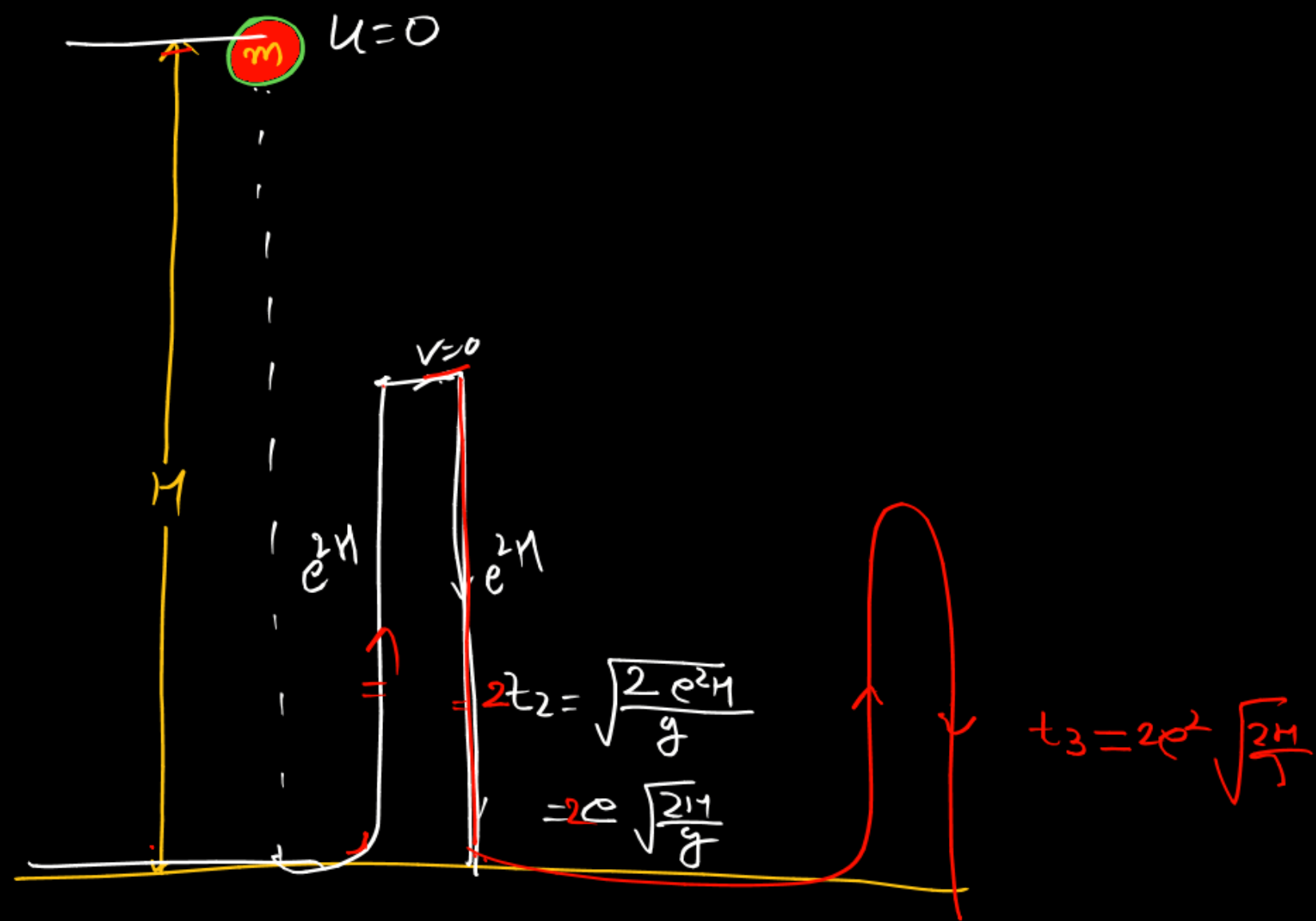
② if $e=1$
 $\text{dist} = \infty$

~~(a) $H \left(\frac{1-e^2}{1+e^2} \right)$~~

~~(b) $H \left(\frac{1-e}{1+e} \right)$~~

~~(c) $\frac{H e^2}{1-e^2}$~~

~~pb~~

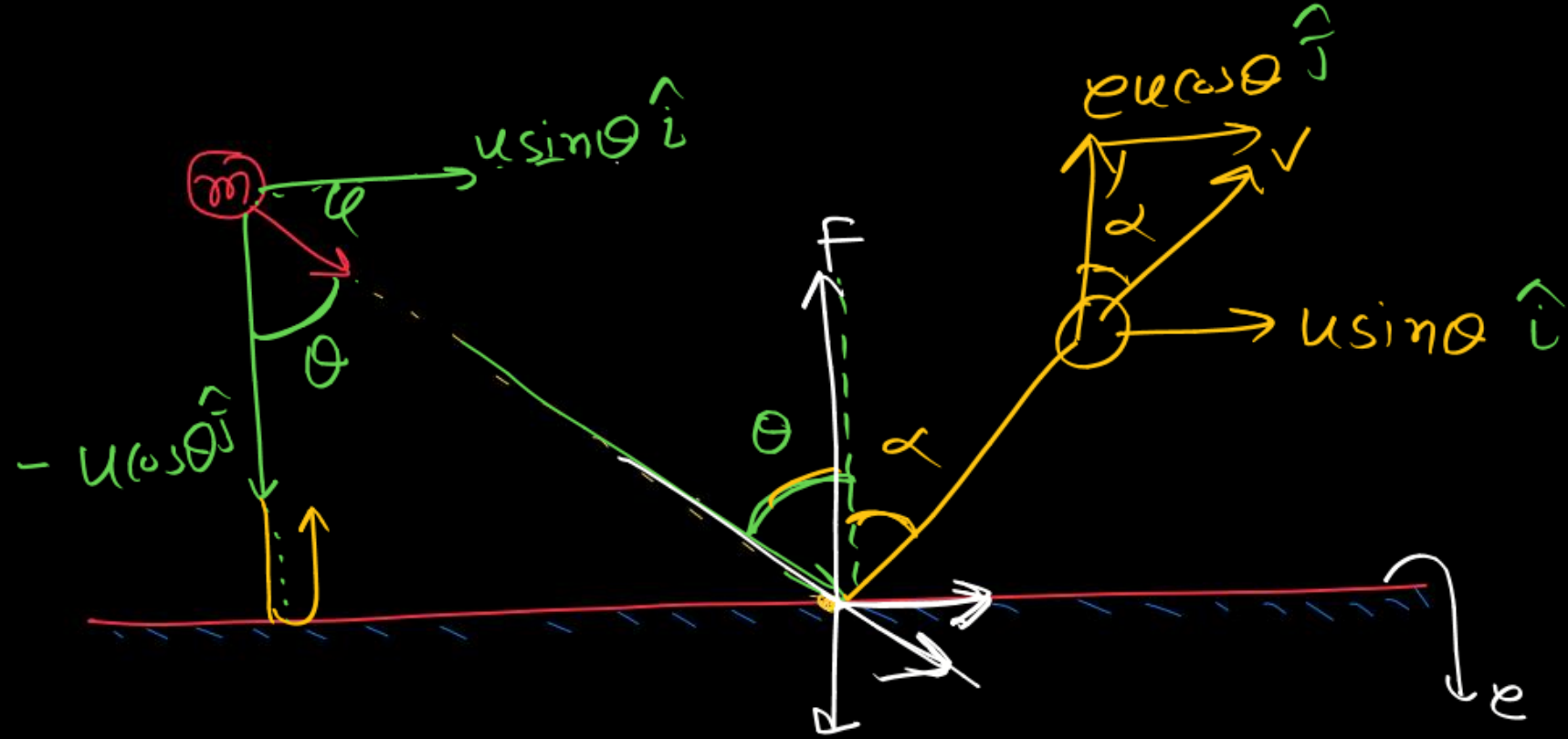


$$t_1 = \sqrt{\frac{2H}{g}}$$

$$T = \sqrt{\frac{2H}{g}} + 2e \sqrt{\frac{2H}{g}} + 2e^2 \sqrt{\frac{2H}{g}} + 2e^3 \sqrt{\frac{2H}{g}}$$

$$= \sqrt{\frac{2H}{g}} \left(\frac{1-e}{1+e} \right)$$

oblique collision



$$\tan \alpha = \frac{u \sin \theta}{e u \cos \theta} = \frac{\tan \theta}{e}$$

$$\vec{V}_{A/c} = u \sin \theta \hat{i} + e u \cos \theta \hat{j}$$

$$\text{Speed } A/c = u \sqrt{\sin^2 \theta + e^2 \cos^2 \theta}$$

MR* Box

velocity parallel to surface do not change. no effect of collision parallel to contact surface.

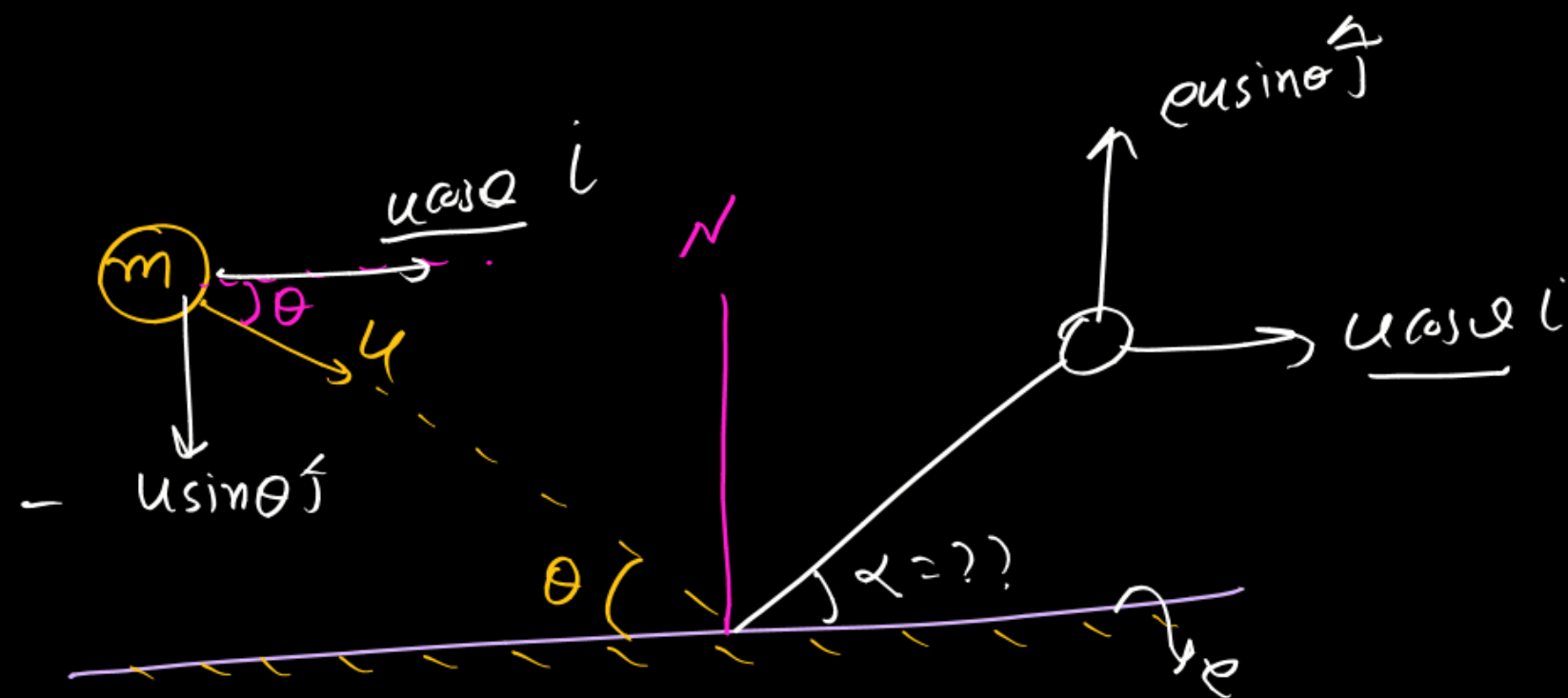
$$u_{A/c} = u \sin \theta \hat{i} - u \cos \theta \hat{j}$$

$$V_{A/c} = u \sin \theta \hat{i} + e u \cos \theta \hat{j}$$

$$\begin{aligned} \vec{V}_{A/c} - \vec{u}_{A/c} &= e u \cos \theta \hat{j} + u \cos \theta \hat{j} \\ &= u \cos \theta (e + 1) \end{aligned}$$

$$\vec{\Delta p} = m (\Delta v) = m u \cos \theta (e + 1)$$

$$I = m u \cos \theta (e + 1)$$



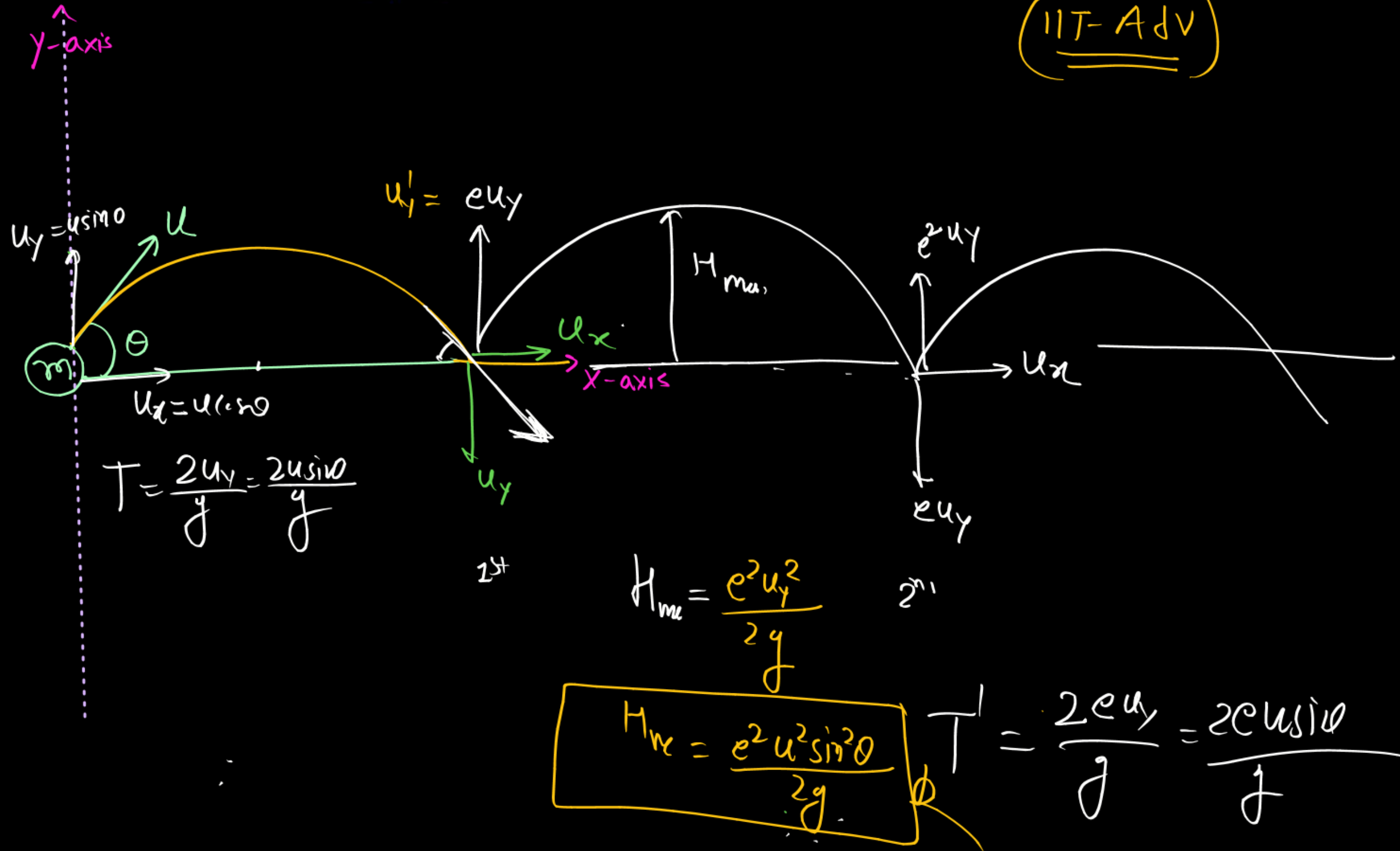
m $\vec{u} = 60\hat{i} - 40\hat{j}$

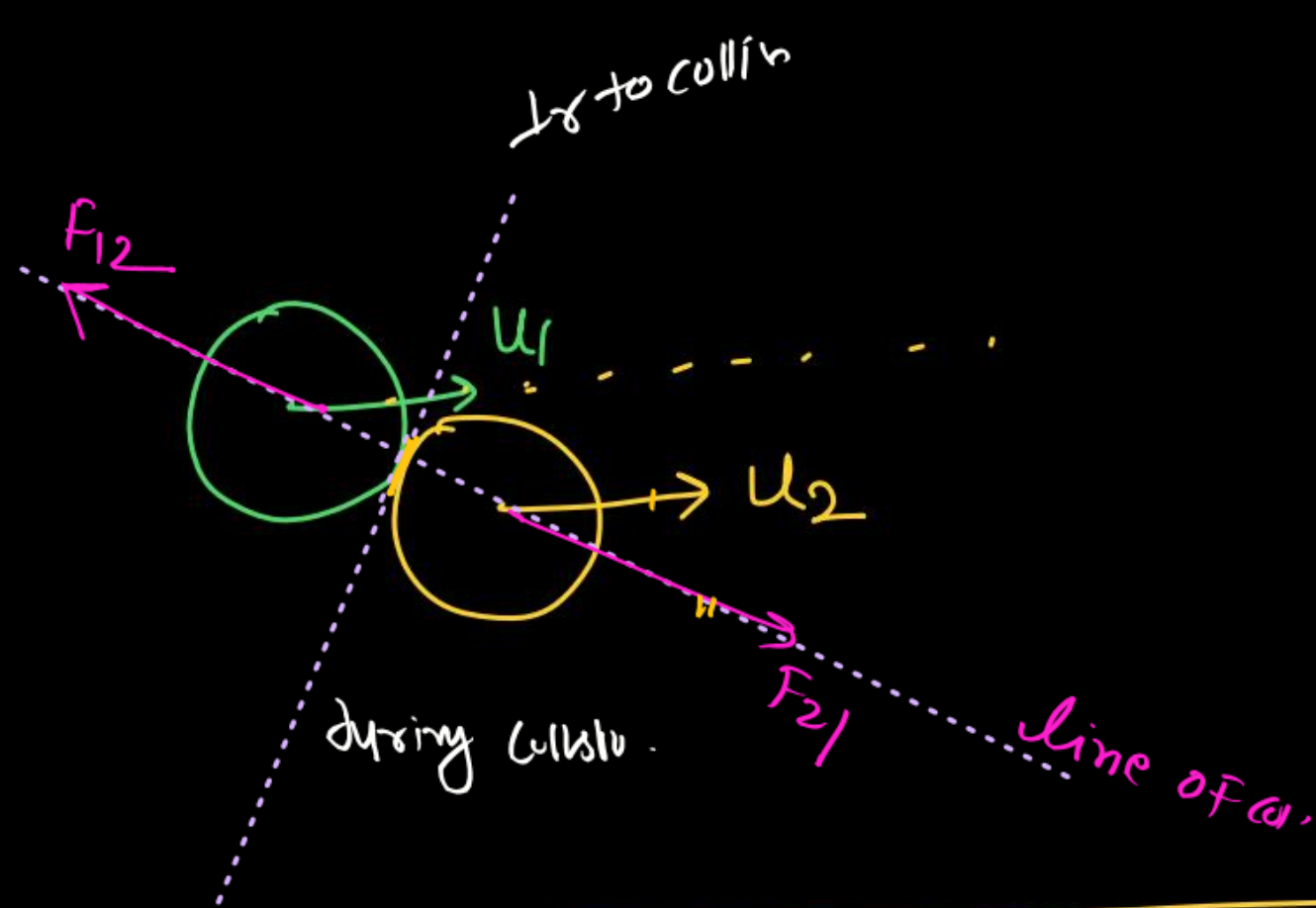
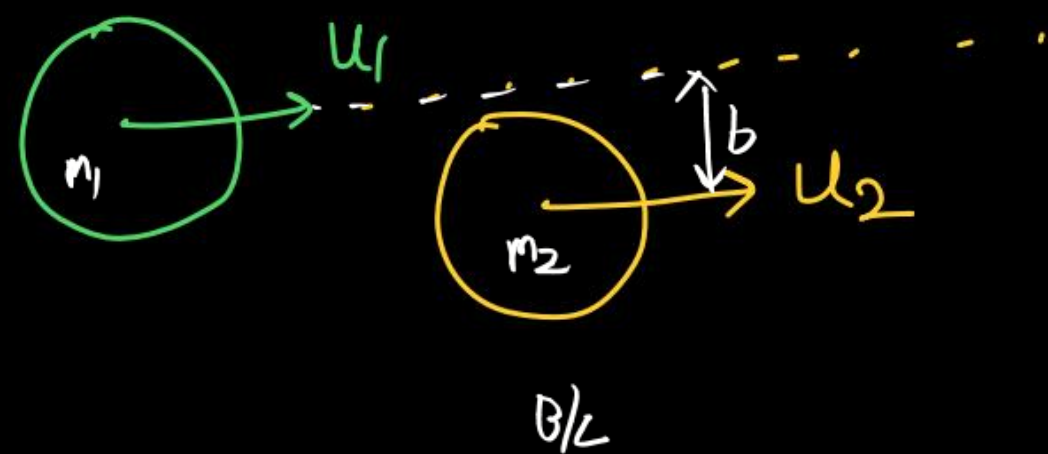
$V_f = ?? = 60\hat{i} + \frac{1}{2}40^{20}\hat{j}$

$e = \frac{1}{2}$

Ball of mass m projected with speed u at an angle ' θ ', coefficient of restitution ' e ' then find $H_{\max}$, T_f B/w 1st and 2nd collision.

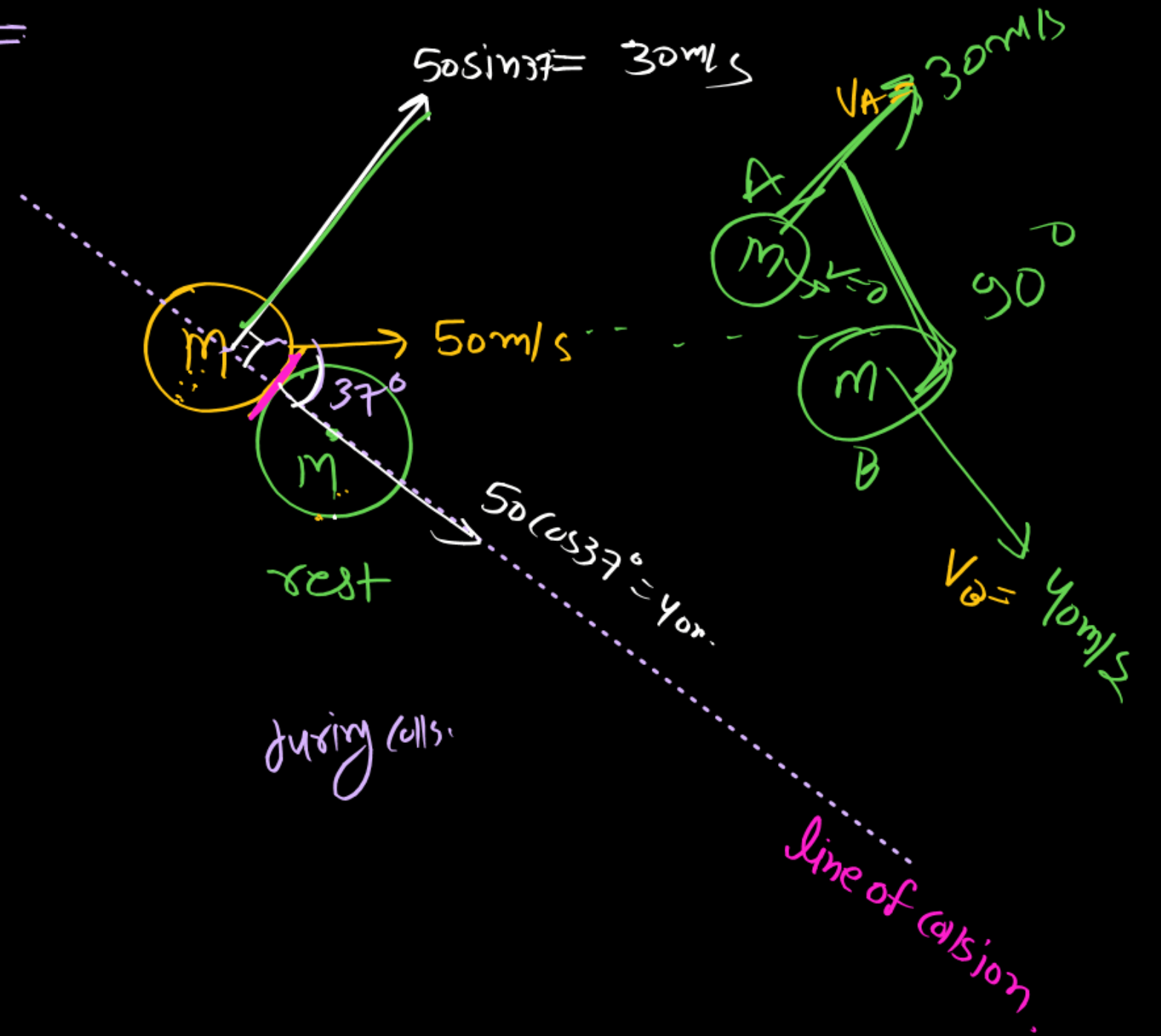
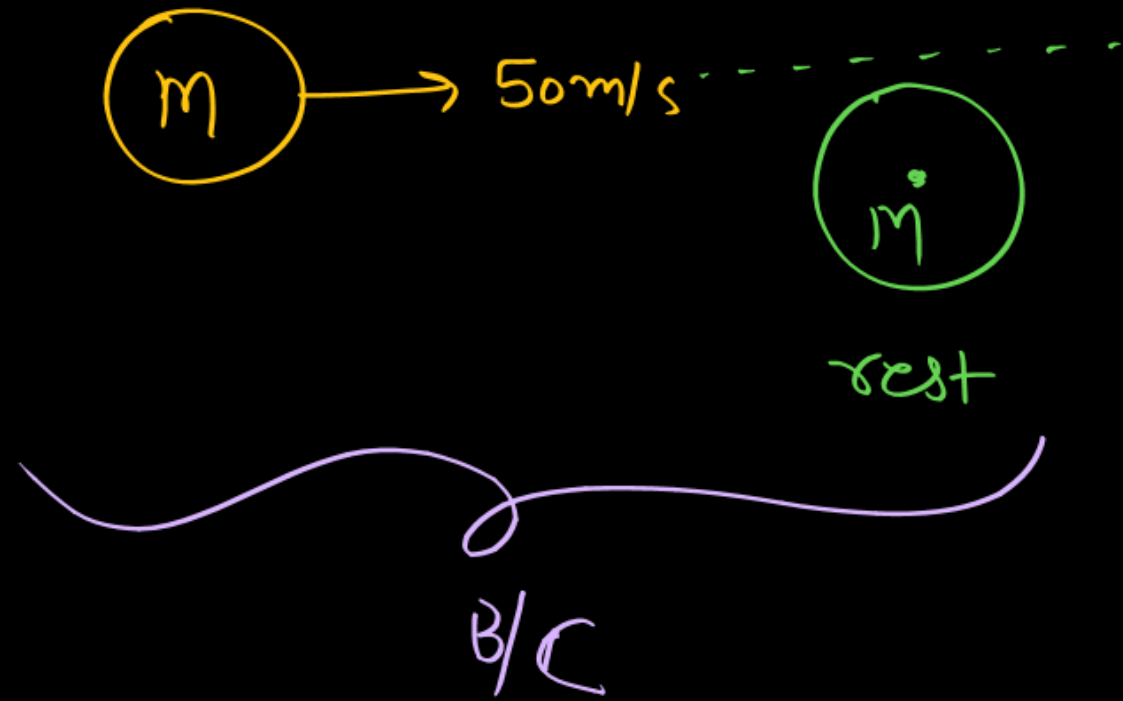
(11T-ADV)



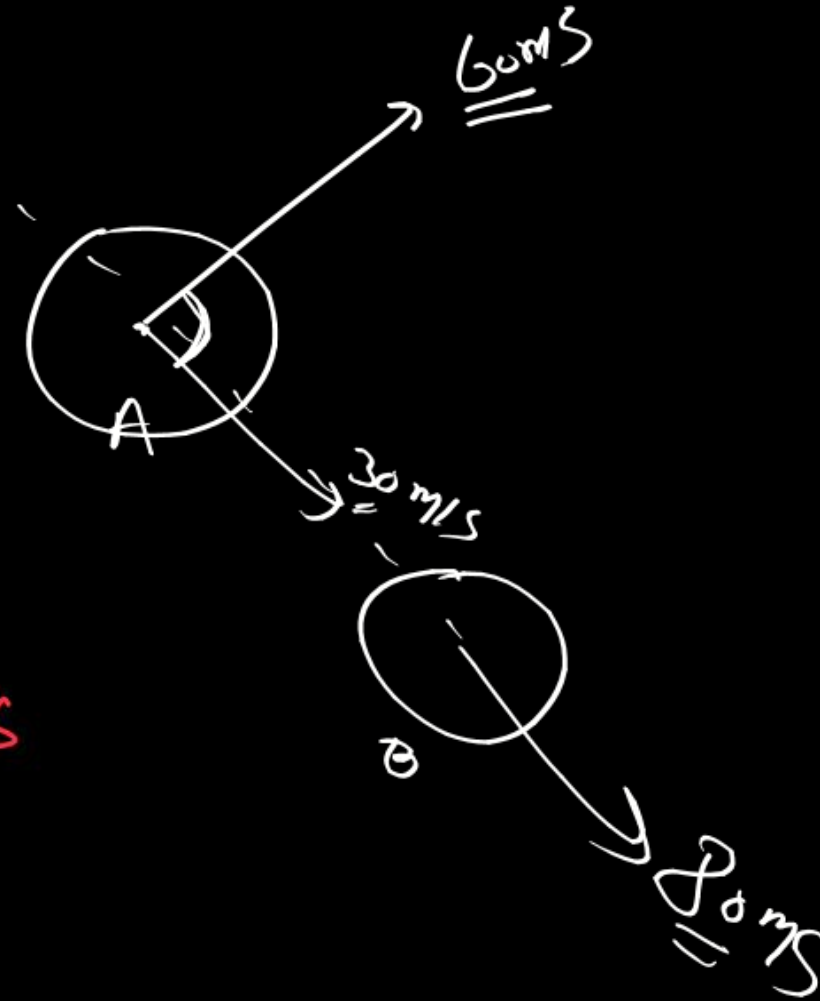
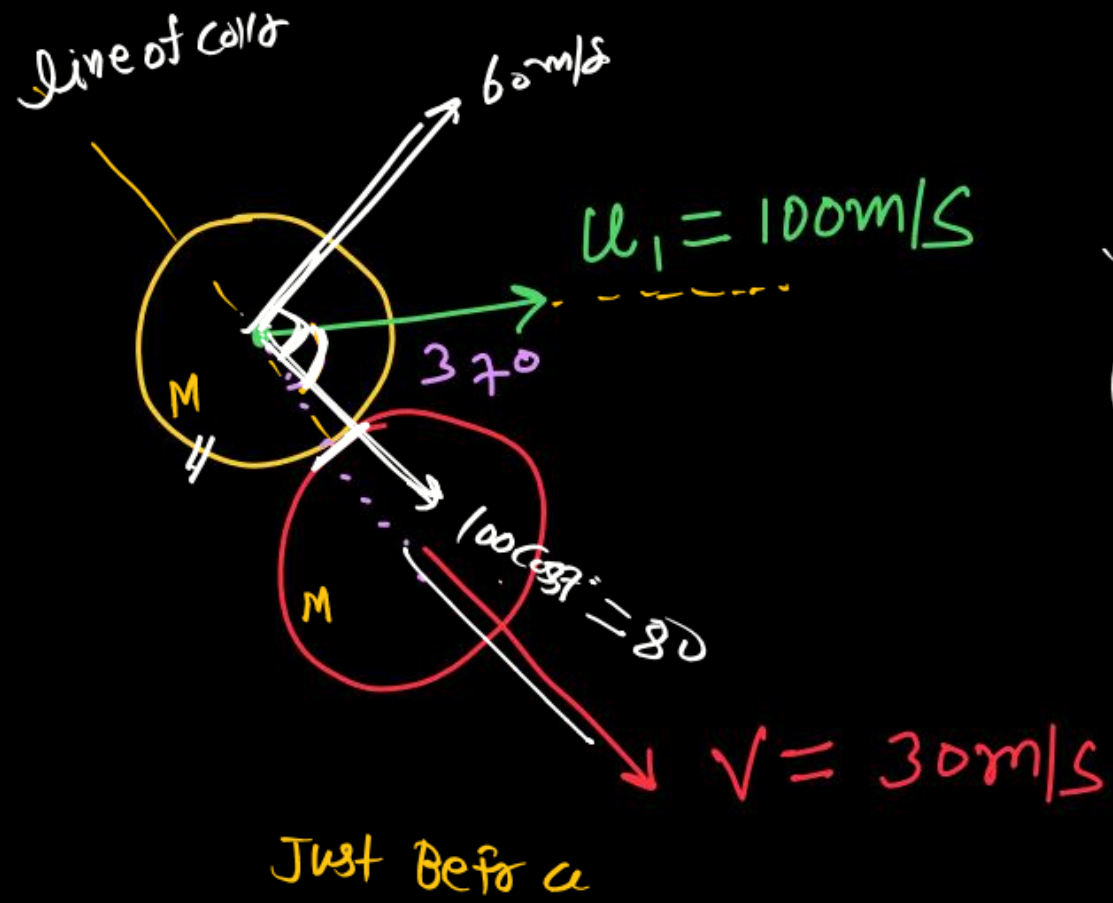


- ✓ MR* Box
Momentum of system conserved in line of collision and perpendicular to line of collision.
- Momentum of a object will not conserved in line of collision.
- Momentum of object conserved $\perp$ line of collision.

find velocity of A & B just A/C
oblique elastic collision of same mass

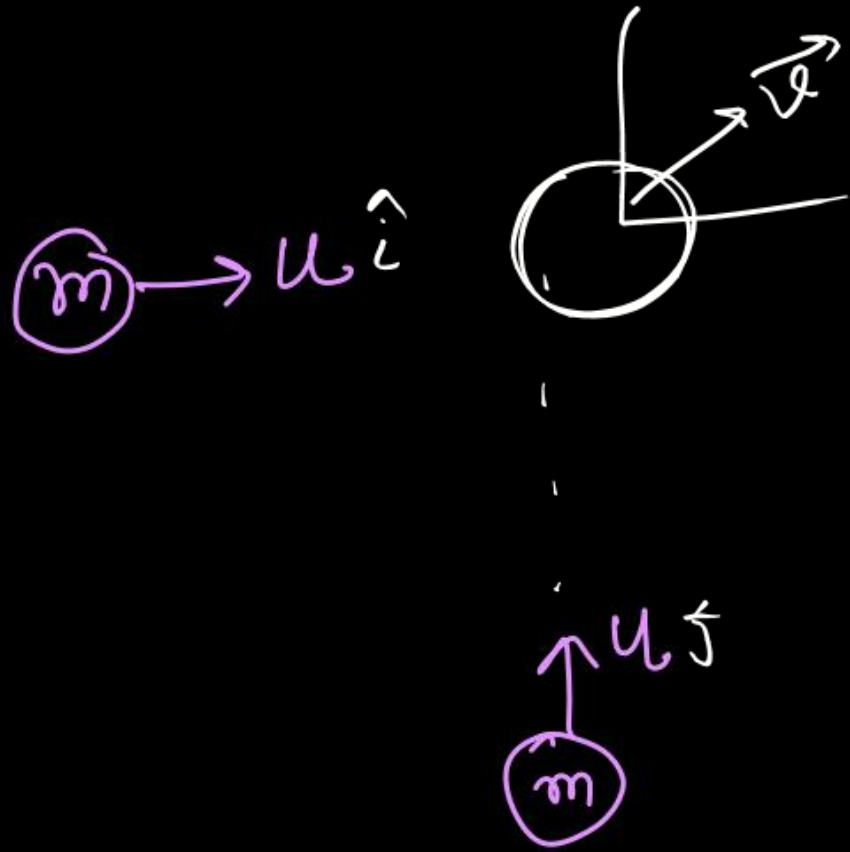


identical object and elastic collision find their final velocity After collision.



elastic collision of same mass
→ velocity will interchange
in line of collision.

Two identical object moving perpendicular to each other if collision is perfectly inelastic then find velocity after collision.



B/c

$$\underline{p_i = p_f}$$
$$mu \hat{i} + mu \hat{j} = 2m \vec{v}$$

$$\boxed{\frac{u \hat{i} + u \hat{j}}{2} = \vec{v}}$$

Question



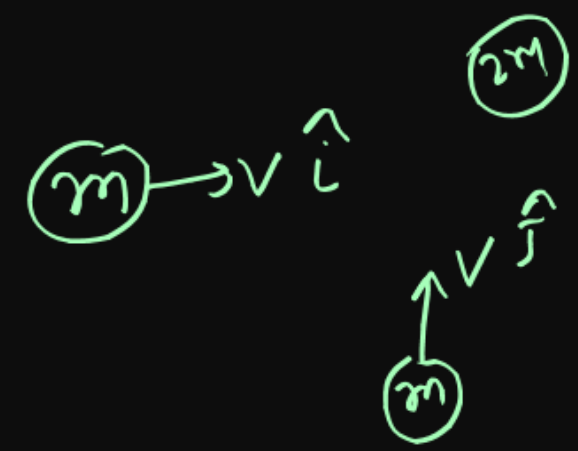
A particle of mass m moving eastward with a speed v collides with another particle of the same mass moving northwards with same speed v . The two particles coalesce on collision. The new particle of mass $2m$ will move with velocity

1 $\frac{v}{2}$ North-East ~~X~~

2 $\frac{v}{\sqrt{2}}$ South-West ~~X~~

3 $\frac{v}{2}$ North-West ~~X~~

4 $\left(\frac{v}{\sqrt{2}}\right)$ North-East ✓



$$p_i = p_f$$
$$mv\mathbf{i} + mv\mathbf{j} = 2mv\mathbf{V}$$
$$v\mathbf{i} + v\mathbf{j} = 2\mathbf{V}$$

$$\mathbf{V} = \frac{v}{2}\mathbf{i} + \frac{v}{2}\mathbf{j}$$

$$|\mathbf{V}| = \frac{v}{2}\sqrt{2} = \frac{v}{\sqrt{2}} \text{ (N-E)}$$

Question



A particle of mass m moving towards west with speed v collides with another particle of mass m moving towards south. If two particles stick to each other, the speed of the new particle of mass $2m$ will be

1 $v\sqrt{2}$

2 $\frac{v}{\sqrt{2}}$

3 $\frac{v}{2}$

4 v

θ/c



cds

Soln $\vec{p}_{ix} = \vec{p}_{fx}$

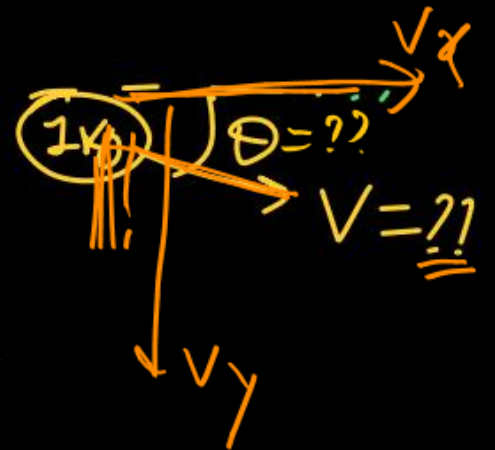
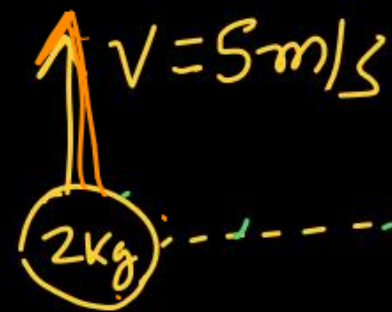
$$1 \times 20 + 2 \times 10 = 0 + 1v_x$$

$$40 = v_x \quad \underline{\underline{v_x = 40 \hat{i}}}$$

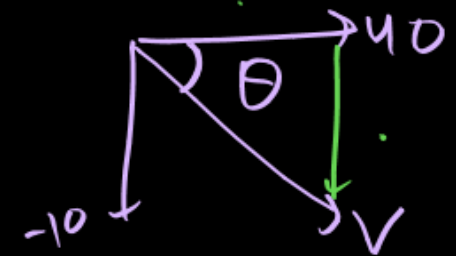
$$p_{iy} = p_{fy}$$

$$0 + 0 = 2 \times 5 \hat{j} + 1v_y$$

$$\boxed{v_y = -10 \hat{j}}$$



$$\vec{v} = 40\hat{i} - 10\hat{j}$$



$$\tan \theta = \frac{10}{40}$$

$$\tan \theta = \frac{1}{4}$$

$$\theta = \tan^{-1}\left(\frac{1}{4}\right)$$

Question

On a frictionless surface, a block of mass M moving at speed v collides elastically with another block of same mass M which is initially at rest. After collision the first block moves at an angle to its initial direction and has a speed $v/3$. The second block's speed after the collision is **[2015]**

1 $\frac{3}{\sqrt{2}}v$

2 $\frac{\sqrt{3}}{2}v$

3 $\frac{2\sqrt{2}}{3}v$

4 $\frac{3}{4}v$



$$K.E_{B/C} = K.E_{A/C}$$

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}m\left(\frac{v}{3}\right)^2 + \frac{1}{2}mv'^2$$

$$v^2 = \frac{v^2}{9} + v'^2$$

$$v'^2 = v^2 - \frac{v^2}{9} = \frac{8v^2}{9}$$

$$v' = \frac{2\sqrt{2}}{3}v$$

mr* box for oblique collision -

→ collision elastic hai & velocity puchha hai to pahle $(K \cdot E)_{Bc} = (K \cdot E)_{Ak}$ try karo then momentum conservation ✓

→ Collision ka Nature known hai, ya Inelastic hai to momentum conserved karo. ✓

→ Kisi bhi collision me Angle (dirⁿ) of velocity pucche to momentum conservation ✓

Question



Two spheres A and B of masses m_1 and m_2 respectively collide. A is at rest initially and B is moving with velocity v along x-axis. After collision B has a velocity $\frac{v}{2}$ in a direction perpendicular to the original direction. The mass A moves after collision in the direction

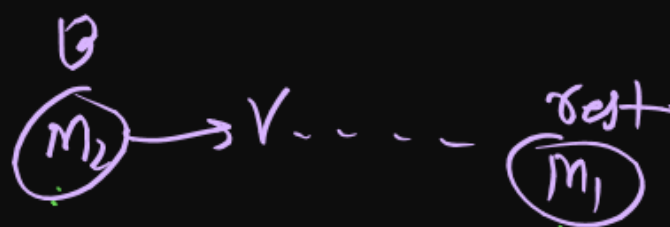
[AIPMT (Prelims)-2012]

1 $\theta = \tan^{-1} \left(\frac{1}{2} \right)$ to the x-axis

2 $\theta = \tan^{-1} \left(-\frac{1}{2} \right)$ to the x-axis

3 Same as that of B

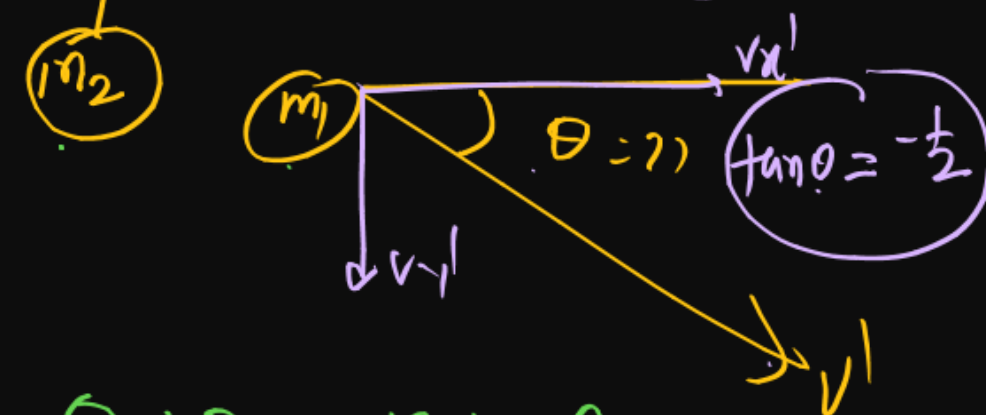
4 Opposite to that of B



$$p_x = \text{con}$$

$$m_2 v = m_1 v_x \quad \text{--- (1)}$$

$$-m_2 \frac{v}{2} = m_1 v_y \quad \text{--- (11)}$$



$$0 + 0 = m_2 \frac{v}{2} \hat{j} + m_1 v_y' \quad \text{--- (11)}$$

$$-\frac{1}{2} = \frac{v_x}{v_y}$$

$$\frac{v_x}{v_y} = -2$$

$$\frac{v_y}{v_x} = -\frac{1}{2}$$

Man + Plank Problem → Ramlal starts motion on plank & move other end of plank, then find distance move by plank.
(No friction) between plank and ground.

$$F_{x(\text{man+plank})}^{\text{ext}} = 0$$

$$F_{\text{ext}} = 0 \quad v_{\text{c.m.}} = \text{const}$$

$$v_{\text{c.m.}} = 0 = v_{\text{c.m. final}}$$

$$\Delta x_{\text{c.m.}} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{(m_1 + m_2)}$$

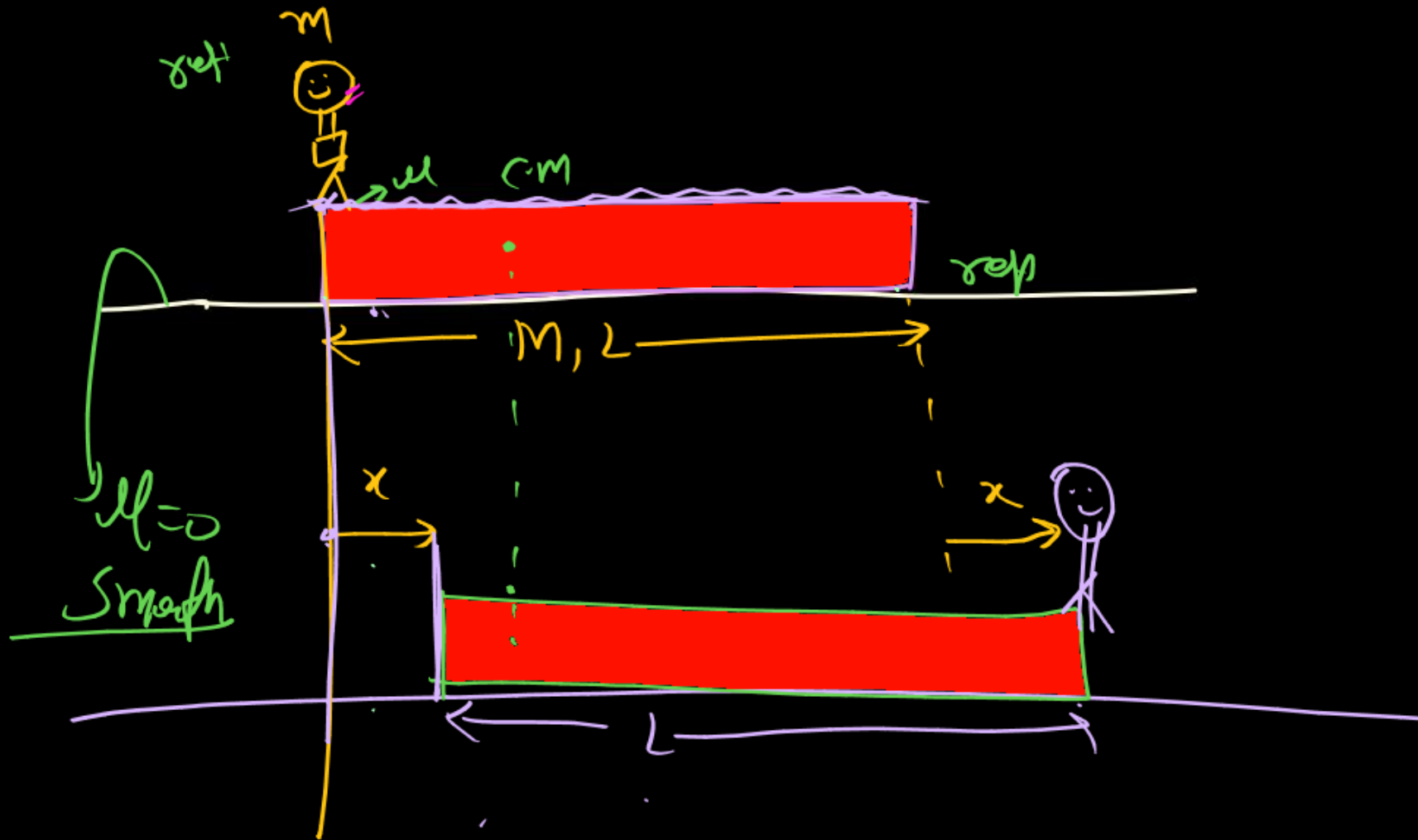
$$m_1 \Delta x_1 + m_2 \Delta x_2 = 0$$

$$m(L+x) + Mx = 0$$

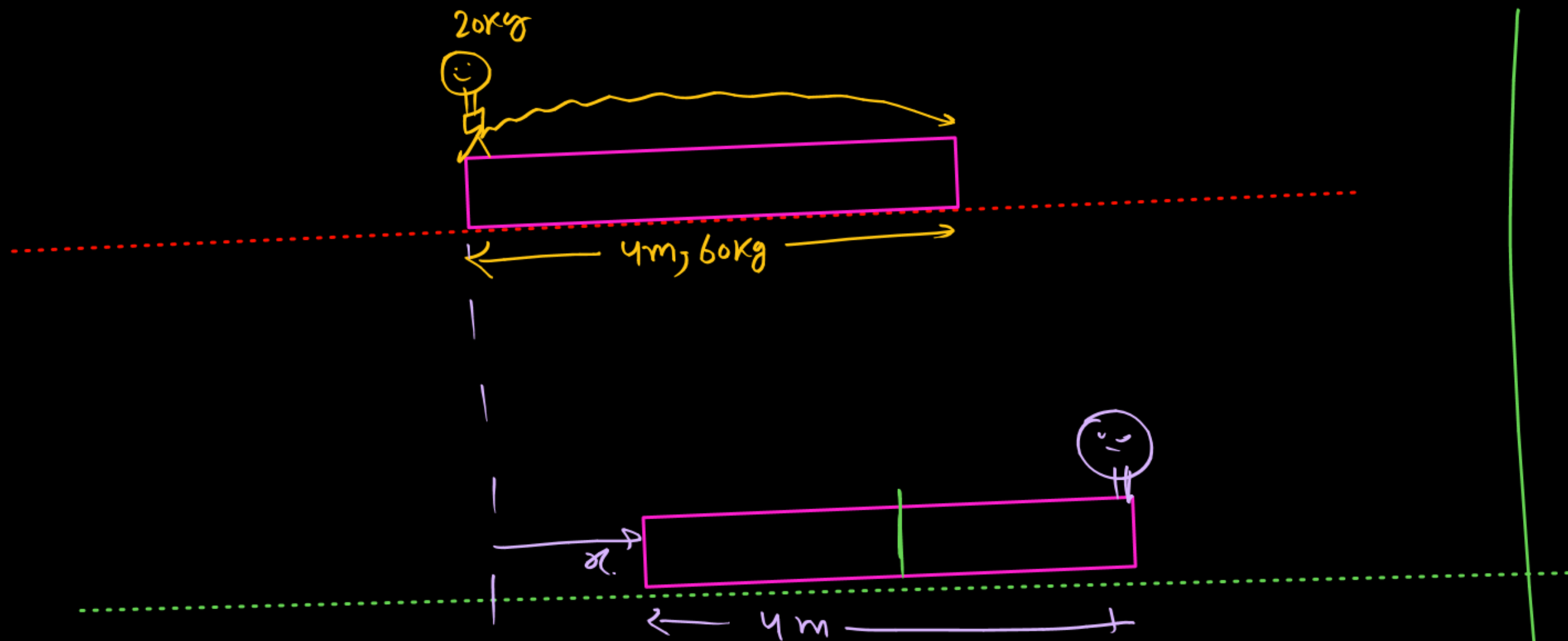
$$mL + mx + Mx = 0$$

$$x(m+M) = -mL$$

$$x = \frac{-mL}{m+M}$$



man walk to other end then find distance moved by plank:-



man walk to mid of plank

$$60x + 20(2+x) = 0$$

$$6x + 4 + 2x = 0$$

$$8x = -4$$

$$x = -\frac{1}{2}m$$

~~$80x = -80$~~

$$20(x+4) + 60(x) = 0$$

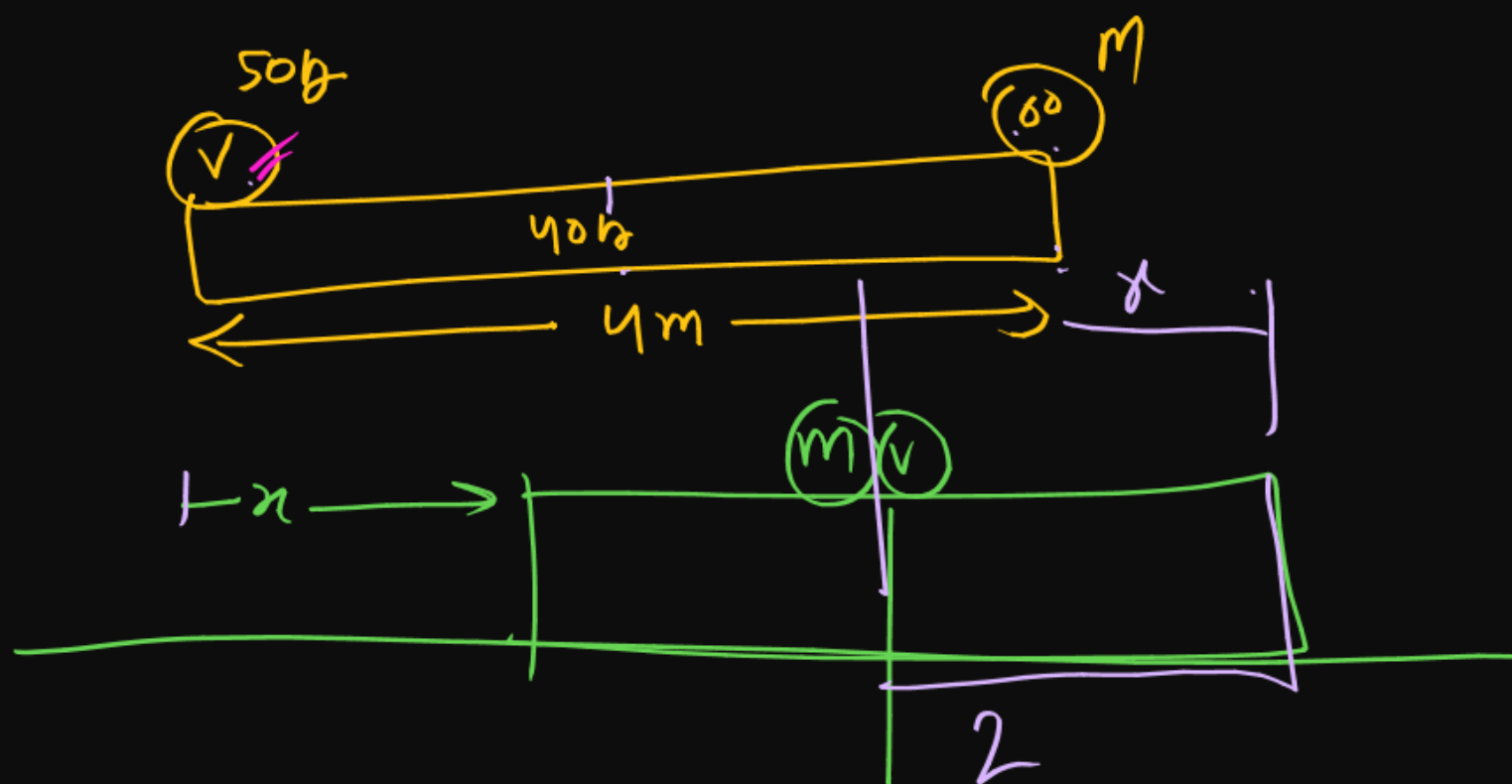
$$20x + 80 + 60x = 0$$

$x = -1m$

Question



Mr. Verma (50 kg) and Mr. Mathur (60 kg) are sitting at the two extremes of a 4 m long boat (40 kg) standing still in water. To discuss a mechanics problem, they come to the middle of the boat. Neglecting friction with water, how far does the boat move on the water during the process?



$$40x + 50(x+2) + 60(2-x) = 0$$

$$4x + 5x + 10 - 12 + 6x = 0$$

$$15x - 2 = 0$$

$$x = +\frac{2}{15}$$

THANK
YOU